Gravitational redshift in quantum-clock interferometry

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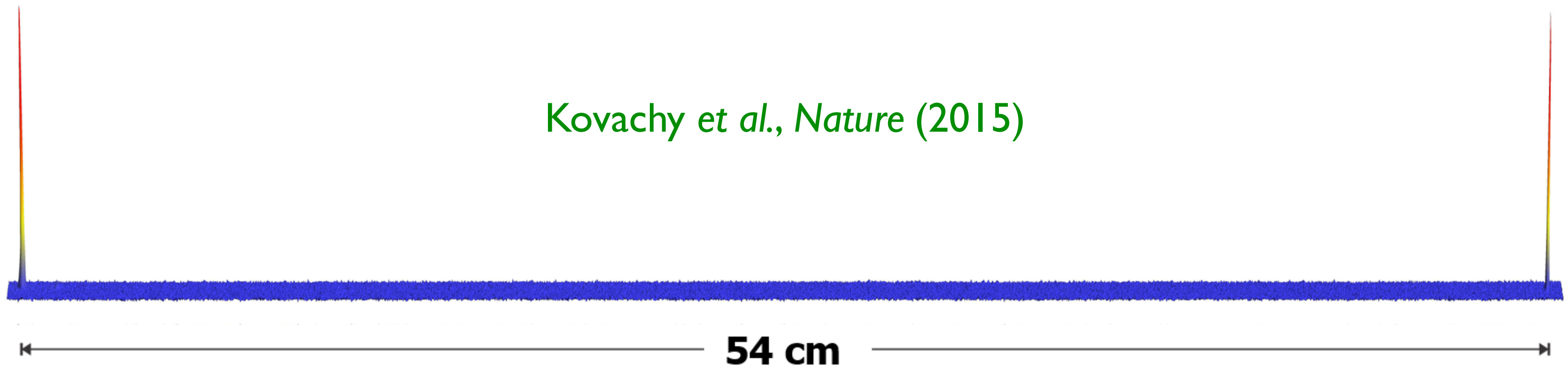
QUANTUS DLR

Relativistic effects in macroscopically delocalized quantum superpositions

- *Macroscopically* delocalized quantum superpositions:

  coherent superposition of atomic wave packets



Kovachy *et al.*, *Nature* (2015)

- Differences in *dynamics* of superposition components

  → entirely Newtonian

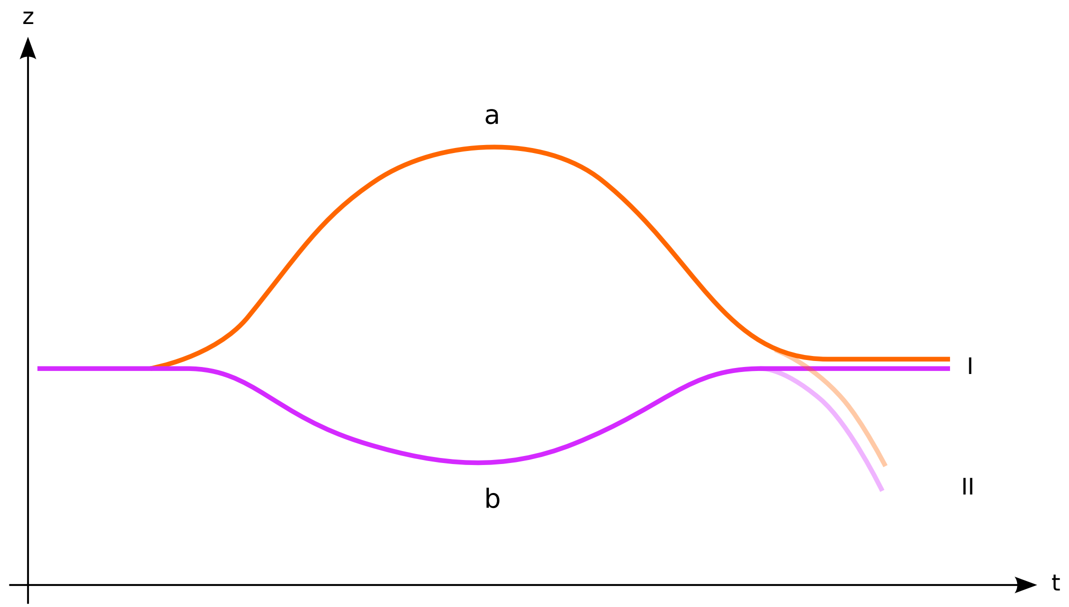
- Same relativistic effects on superposition components

  (e.g. atomic clocks)

★ <u>Goal</u> (QM + GR): experiment with general relativistic effects acting *non-trivially* on the quantum superposition

Proper time as *which-way* information

- Quantum superposition of clocks (*COM* + *internal state*) experiencing different proper times



$|\psi_a\rangle + |\psi_b\rangle$

→ *reduced* *visibility* of interference signal

Zych *et al.*, *Nat. Comm.* (2011)

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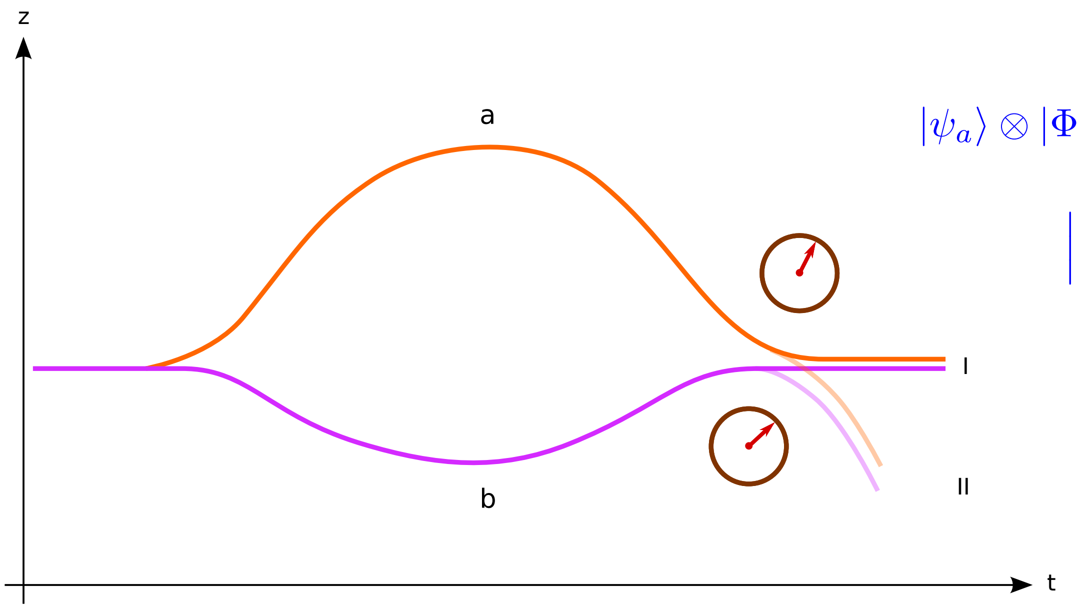


$$|\psi_a\rangle \otimes |\Phi(\tau_a)\rangle + |\psi_b\rangle \otimes |\Phi(\tau_b)\rangle$$

$$\Big|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\Big| < 1$$

→ *reduced* *visibility* of interference signal

Zych *et al.*, *Nat. Comm.* (2011)

Outline

1. Relativistic effects in macroscopically delocalized quantum superpositions

2. Key elements of *quantum-clock interferometry*

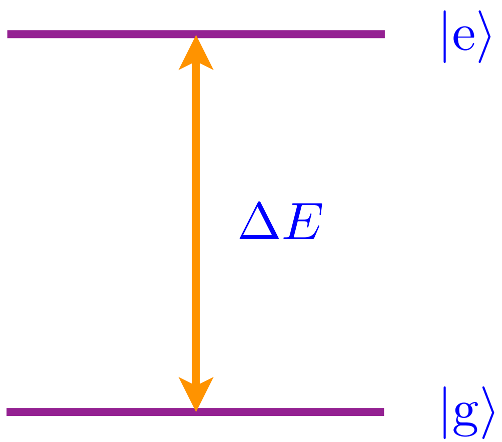
3. Major challenges in quantum-clock interferometry

4. *Doubly differential* scheme for *gravitational-redshift* measurements

5. Feasibility and extensions

Key elements of quantum-clock interferometry

Quantum-clock model



- *Initialization* pulse:

$$|\mathrm{g}\rangle \;\to\; \big|\Phi(0)\big\rangle = \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\, e^{i\varphi}|\mathrm{e}\rangle\Big)$$

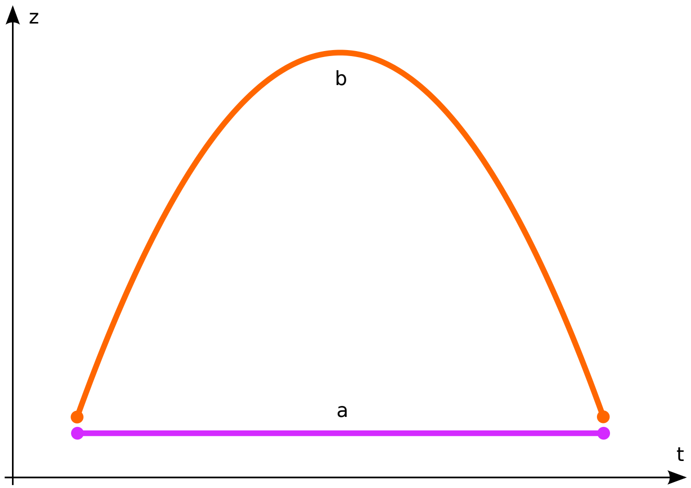
- Evolution:

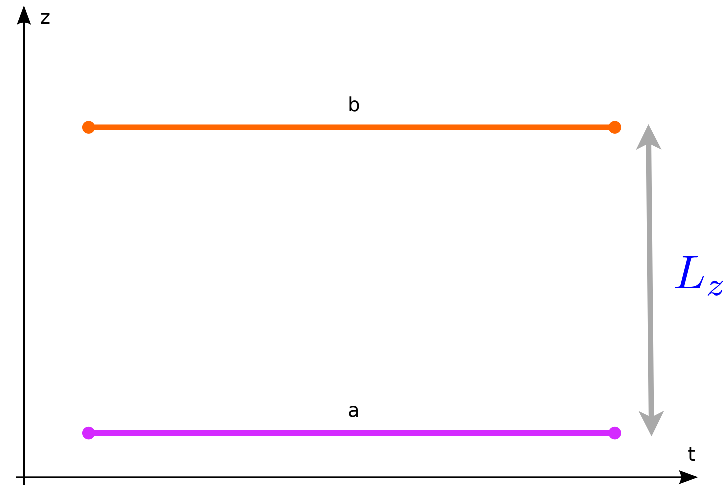
$$\big|\Phi(\tau)\big\rangle \propto \frac{1}{\sqrt{2}}\Big(|\mathrm{g}\rangle + i\, e^{i\varphi} e^{-i\Delta E\,\tau/\hbar}|\mathrm{e}\rangle\Big)$$

- Quantum overlap:

$$\Big|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\Big| = \cos\left(\frac{\Delta E}{2\hbar}\,(\tau_b - \tau_a)\right)$$

- Comparison of independent clocks (after *read-out* pulse):





$$\Delta\tau_b - \Delta\tau_a \approx \left(g\,L_z/c^2\right)\Delta t$$

for optical *atomic clocks*

$\Delta E \sim 1\,\mathrm{eV}$ $\quad L_z \sim 1\,\mathrm{cm}$

- Instead of independent clocks we pursue a quantum superposition at *different heights*.

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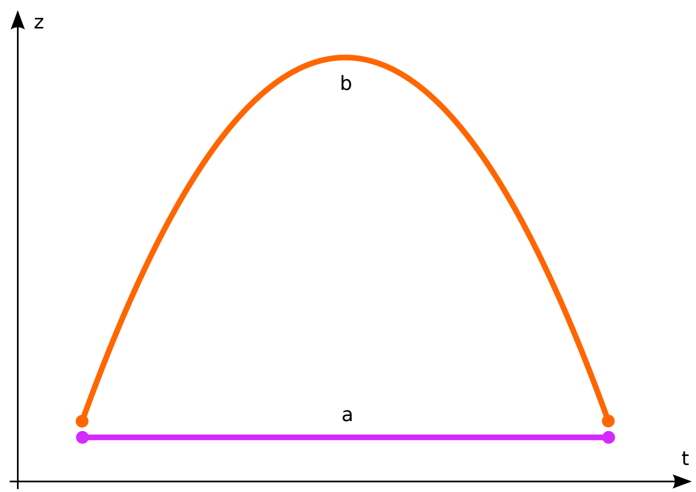

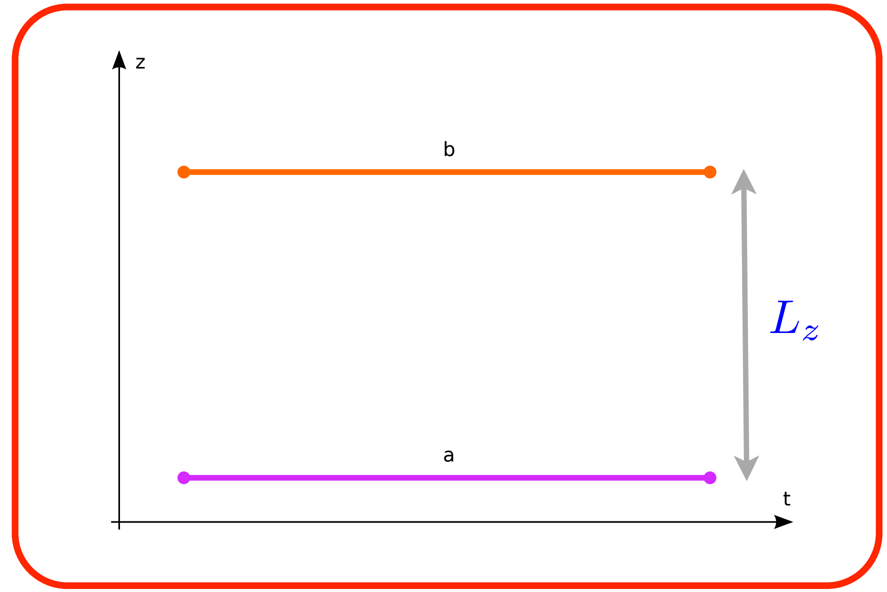

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- Instead of independent clocks we pursue a quantum superposition at *different heights*.

- Theoretical description of the clock

  - *two-level* atom (*internal* state):

$$\hat{H} = \hat{H}_1 \otimes |\mathrm{g}\rangle\langle\mathrm{g}| + \hat{H}_2 \otimes |\mathrm{e}\rangle\langle\mathrm{e}|$$

$$m_1 = m_{\mathrm{g}}$$

$$m_2 = m_{\mathrm{g}} + \Delta m$$

$$\Delta m = \Delta E / c^2$$

  - classical action for *COM* motion:

$$S_n[x^\mu(\lambda)] = -m_n c^2 \int d\tau = -m_n c \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \qquad (\, n = 1, 2 \,)$$

free fall

$$S_n[x^\mu(\lambda)] = -m_n c^2 \int d\tau - \int d\tau \, V_n(x^\mu)$$

including external forces

Atom interferometry in curved spacetime (including relativistic effects)

- Wave-packet evolution in terms of
  - *central trajectory* (satisfies *classical e.o.m.*) $X^\mu(\lambda)$
  - *centered* wave packet $|\psi_{\mathrm{c}}^{(n)}(\tau_{\mathrm{c}})\rangle$

- Fermi-Walker frame associated with the *central trajectory*
  - valid for *freely falling* wave packet (*geodesic*)
  - but also with *external forces / guiding* potential (*accel.* trajectory)
  - approximately *non-relativistic* dynamics for centered wave packet

$\Delta p/m \ll c$ $\Delta x \ll \ell$

curvature radius

- Metric in *Fermi-Walker* coordinates: $X^{\mu}(\tau_{\mathrm{c}}) = \big(c\,\tau_{\mathrm{c}}\,,\mathbf{0}\big)$

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = g_{00}\, c^2 d\tau_{\mathrm{c}}^2 \,+\, 2\, g_{0i}\, c\, d\tau_{\mathrm{c}}\, dx^i \,+\, g_{ij}\, dx^i dx^j$$

$$\begin{aligned}
g_{00} &= -\big(1 + \delta_{ij}\, a^i(\tau_{\mathrm{c}})\, x^j / c^2\,\big)^2 - R_{0i0j}(\tau_{\mathrm{c}}, \mathbf{0})\, x^i x^j + O\big(|\mathbf{x}|^3\big) \\
g_{0i} &= -\frac{2}{3} R_{0jik}(\tau_{\mathrm{c}}, \mathbf{0})\, x^j x^k \,+\, O\big(|\mathbf{x}|^3\big) \\
g_{ij} &= \delta_{ij} - \frac{1}{3} R_{ikjl}(\tau_{\mathrm{c}}, \mathbf{0})\, x^k x^l \,+\, O\big(|\mathbf{x}|^3\big)
\end{aligned}$$

- Expanding the action for the *centered wave packet:*

$$S_n\big[\mathbf{x}(t)\big] \,\approx\, \int d\tau_{\mathrm{c}} \Big[-m_n c^2 - V_n(\tau_{\mathrm{c}}, \mathbf{0}) + \frac{m_n}{2}\, \mathbf{v}^2 - \frac{1}{2}\, \mathbf{x}^{\mathrm{T}} \Big(\mathcal{V}^{(n)}(\tau_{\mathrm{c}}) - m_n \Gamma(\tau_{\mathrm{c}}) \Big)\, \mathbf{x} \,-\, V^{(n)}_{\mathrm{anh.}}(\tau_{\mathrm{c}}, \mathbf{x}) \Big]$$

- Hamiltonian: $$\hat{H}_n = m_n c^2 + V_n(\tau_\mathrm{c}, \mathbf{0}) + \hat{H}_\mathrm{c}^{(n)}$$

$$\mathcal{V}_{ij}^{(n)}(\tau_\mathrm{c}) = \partial_i \partial_j V_n(\tau_\mathrm{c}, \mathbf{x})\big|_{\mathbf{x}=\mathbf{0}}$$

$$\hat{H}_\mathrm{c}^{(n)} = \frac{1}{2m_n}\,\hat{\mathbf{p}}^2 + \frac{1}{2}\,\hat{\mathbf{x}}^\mathrm{T}\Big(\mathcal{V}^{(n)}(\tau_\mathrm{c}) - m_n \Gamma(\tau_\mathrm{c})\Big)\,\hat{\mathbf{x}}$$

- Wave-packet evolution: $$\big|\psi^{(n)}(\tau_\mathrm{c})\big\rangle = e^{i\,\mathcal{S}_n/\hbar}\,\big|\psi_\mathrm{c}^{(n)}(\tau_\mathrm{c})\big\rangle$$

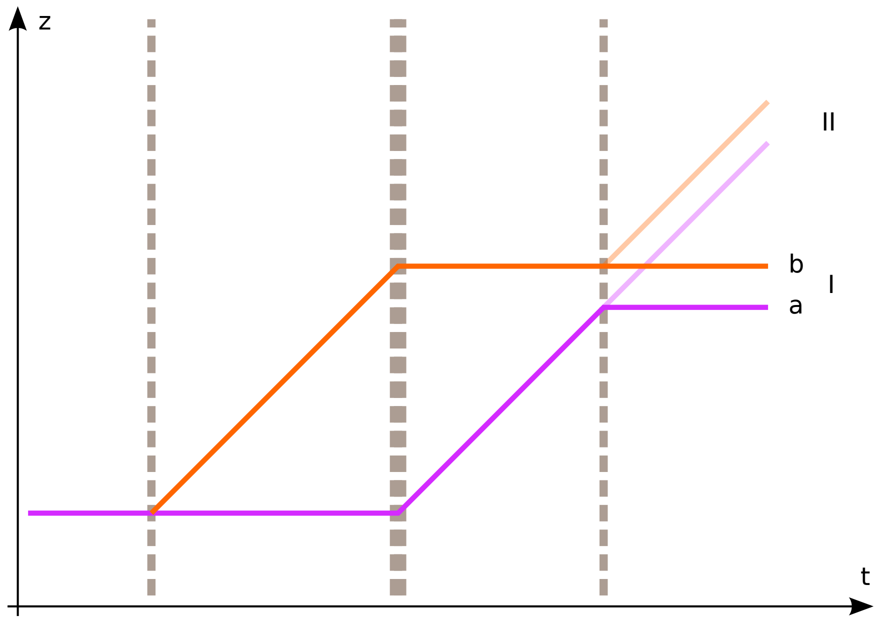
  - *propagation phase*

$$\mathcal{S}_n = -\int_{\tau_1}^{\tau_2} d\tau_\mathrm{c}\,\left(m_n c^2 + V_n(\tau_\mathrm{c}, \mathbf{0})\right)$$

  - *centered wave packet*

$$i\hbar\,\frac{d}{d\tau_\mathrm{c}}\big|\psi_\mathrm{c}^{(n)}(\tau_\mathrm{c})\big\rangle = \hat{H}_\mathrm{c}\,\big|\psi_\mathrm{c}^{(n)}(\tau_\mathrm{c})\big\rangle$$

- Full interferometer (including *laser kicks*):



propagation + *laser* phases

$$|\psi_{\rm I}\rangle = \frac{1}{2}\left(e^{i\phi_a} + e^{i\phi_b}\right)|\psi_{\rm c}\rangle$$

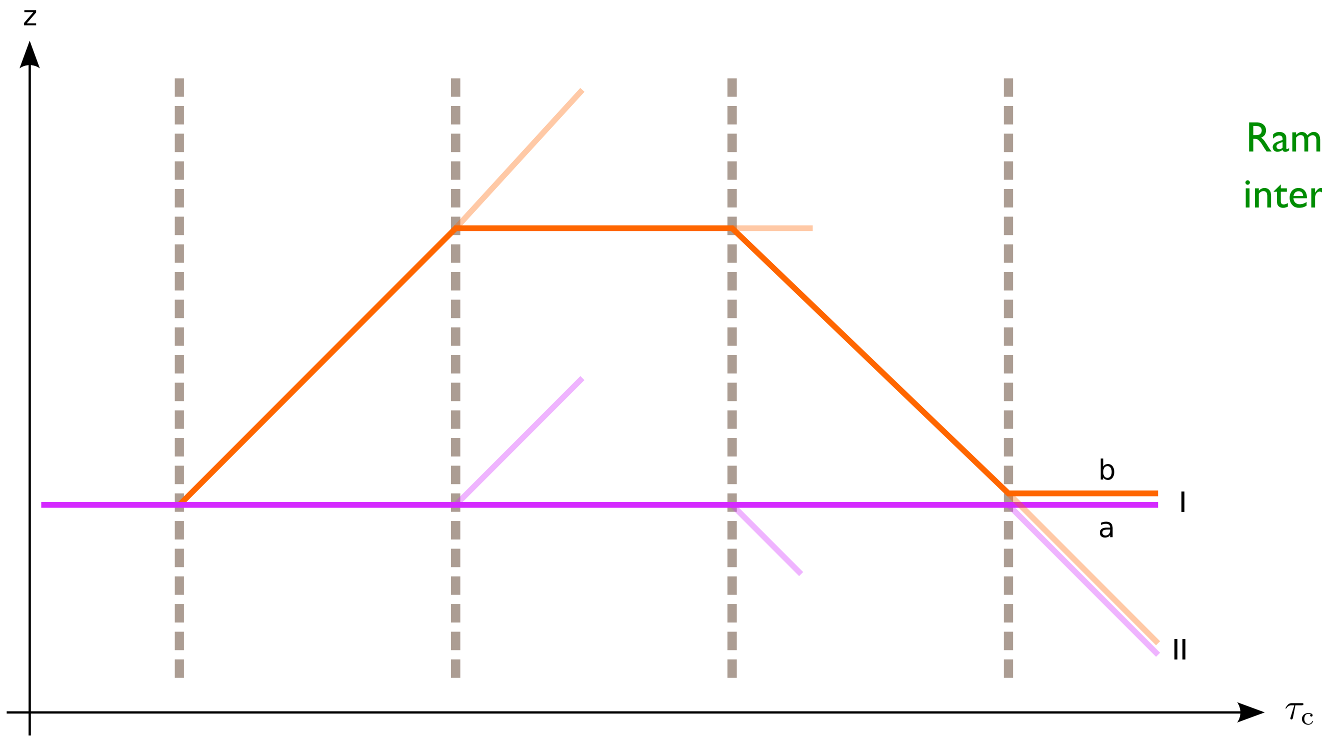
- Detection *probability* at the *exit port(s)*: $\langle\psi_{\rm I}|\psi_{\rm I}\rangle = \dfrac{1}{2}\left(1+\cos\delta\phi\right)$

- *Phase shift:* $\delta\phi = \phi_b - \phi_a + \delta\phi_{\rm sep}$

Major challenges
in quantum-clock interferometry

Insensitivity to gravitational redshift (in a *uniform field*)

- Consider a freely falling frame:



- Proper-time difference between the two interferometer branches $\longrightarrow$ independent of g

(small dependence due to pulse timing suppressed by $(v_{\rm rec}/c) \sim 10^{-10}$)

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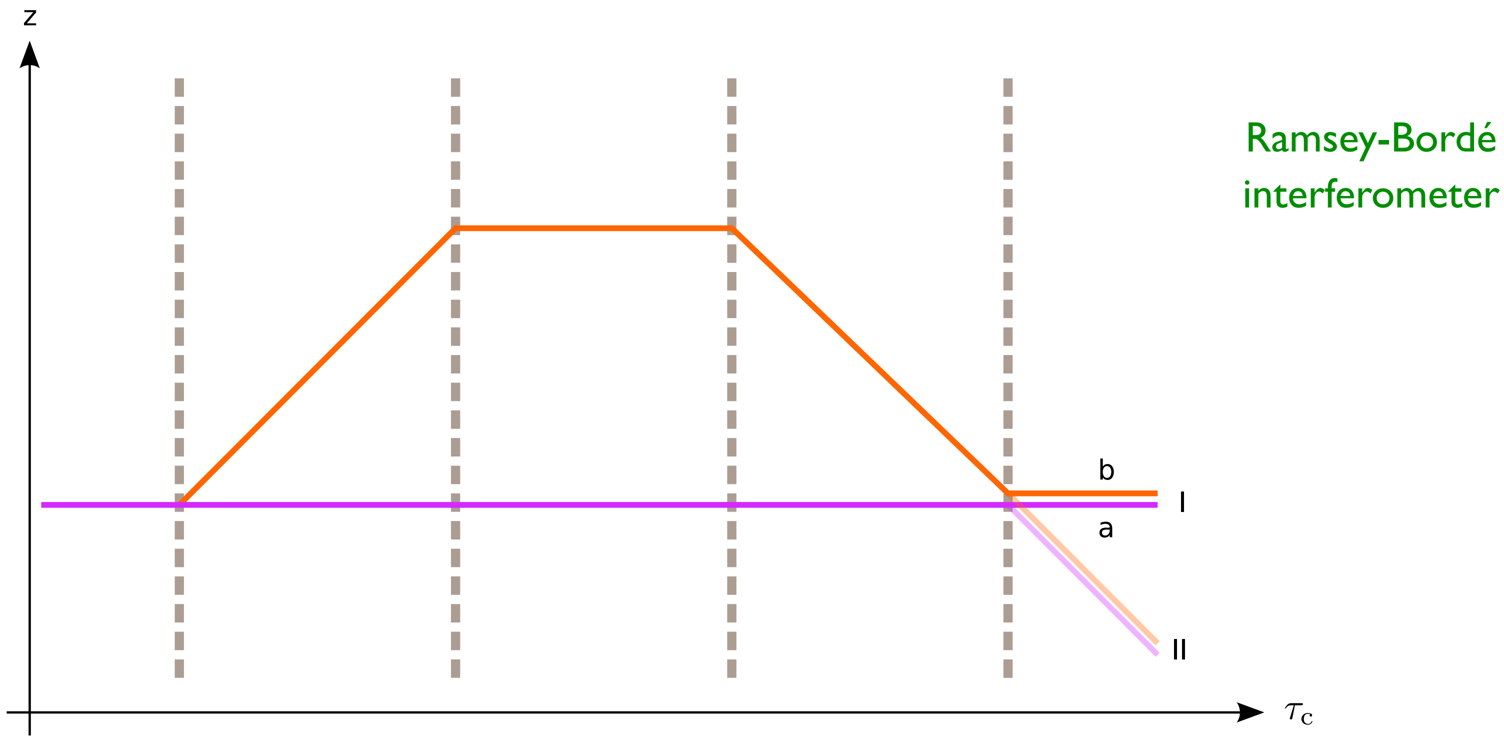

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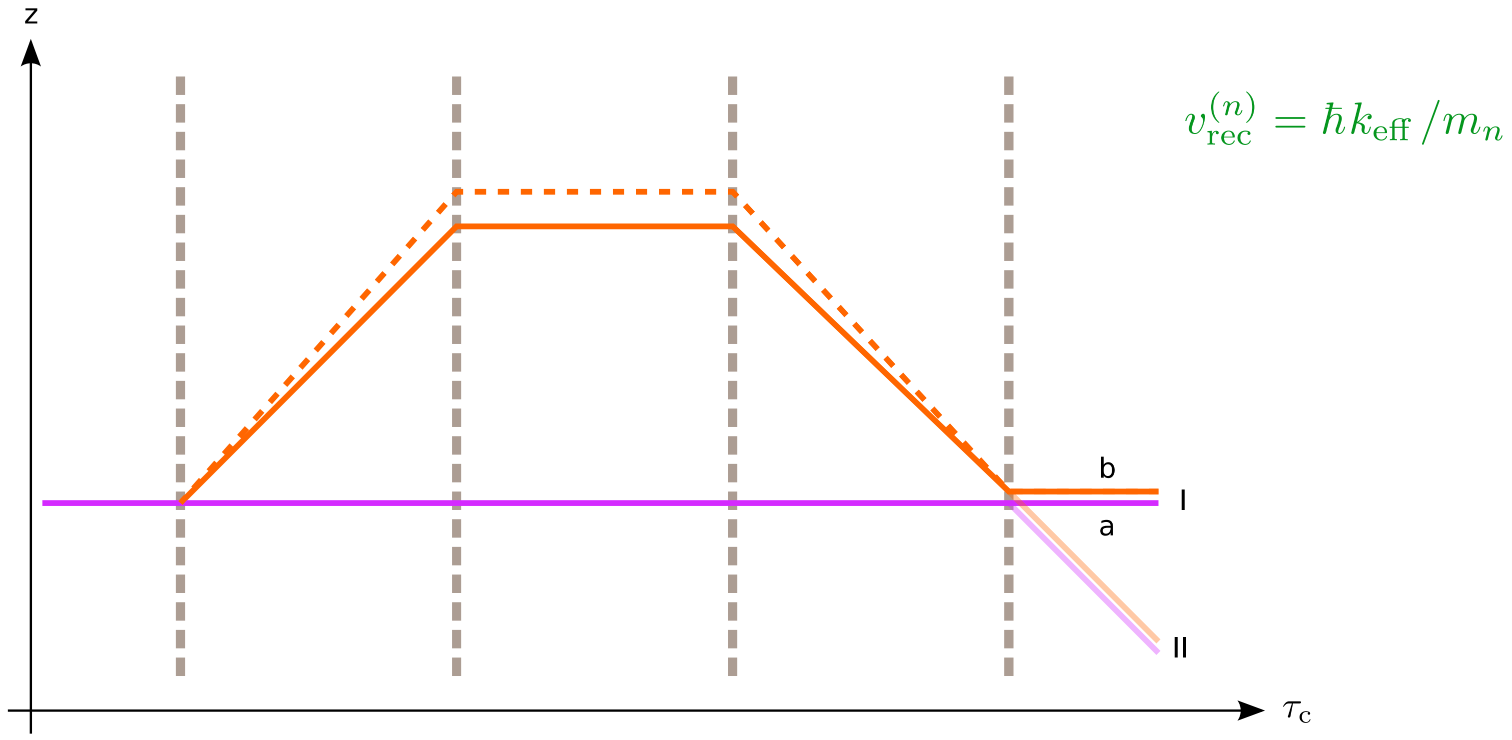
Differential recoil

- Different *recoil velocities* → different *central trajectories*



- Implied changes of proper-time difference are comparable to signal of interest.

Small visibility changes

- Reduced interference visibility due to deceasing quantum overlap of *clock states*:

$$\langle\Psi_{\mathrm{I}}|\Psi_{\mathrm{I}}\rangle = \frac{1}{2} + \frac{1}{2}\left|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\right| \cos\delta\phi$$

$$\left|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\right| = \cos\left(\frac{\Delta E}{2\hbar}\left(\tau_b - \tau_a\right)\right)$$

- Small effect for feasible parameter range:

$$\left|\langle\Phi(\tau_b)|\Phi(\tau_a)\rangle\right| = \cos\left(\frac{\omega_0}{2}\,\frac{g\,\Delta z}{c^2}\,\Delta t\right) \approx 1 - \left(10^{-3}\right)^2/2$$

$$\Delta E/\hbar = \omega_0 \approx 2\pi \times 4 \times 10^2\,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{cm}$$

$$\Delta t = 1\,\mathrm{s}$$

- Extremely difficult to measure such small *changes of visibility*, which are masked by *other effects* leading also to *loss of visibility*.

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$$\langle \Psi_{\mathrm{I}} | \Psi_{\mathrm{I}} \rangle = \frac{1}{2} + \frac{1}{2} \Big| \langle \Phi(\tau_b) | \Phi(\tau_a) \rangle \Big| \cos \delta\phi$$

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$$\Delta E / \hbar = \omega_0 \approx 2\pi \times 4 \times 10^2 \,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{m}$$

$$\Delta t = 1\,\mathrm{s}$$

- Extremely difficult to measure such small *changes of visibility*, which are masked by *other effects* leading also to *loss of visibility*.

Doubly differential scheme
for gravitational-redshift measurement

Differential phase-shift measurement

- Detection probability at first exit port (*independent* of *internal state*):

$$\begin{aligned}
\langle \Psi_{\mathrm{I}} | \Psi_{\mathrm{I}} \rangle &= \frac{1}{2} \left(\langle \Psi_{\mathrm{I}}^{(1)} | \Psi_{\mathrm{I}}^{(1)} \rangle + \langle \Psi_{\mathrm{I}}^{(2)} | \Psi_{\mathrm{I}}^{(2)} \rangle \right) \\
&= \frac{1}{4} \left(2 + \cos \delta\phi^{(1)} + \cos \delta\phi^{(2)} \right) \\
&= \frac{1}{2} + \underbrace{\frac{1}{2} \cos \left(\frac{\delta\phi^{(2)} - \delta\phi^{(1)}}{2} \right)}_{\text{visibility}} \cos \left(\frac{\delta\phi^{(1)} + \delta\phi^{(2)}}{2} \right)
\end{aligned}$$

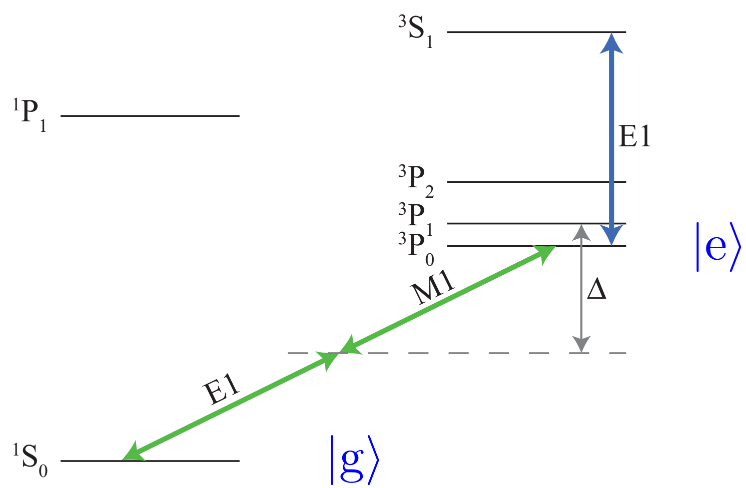
- Phase-shift difference directly related to visibility reduction.

- Precise differential phase-shift measurement involving *state-selective detection* is much more viable.

  (*immune* to spurious loss of contrast + *common-mode rejection* of phase noise)

Two-photon pulse for clock initialization

- Level structure for group-II-type atoms (e.g. *Sr*, *Yb*) employed in optical atomic clocks:



Alden *et al.*, *Phys. Rev. A* (2014)

- Two-photon process *resonantly* connecting the two clock states.

- Equal-frequency *counter-propagating* laser beams in lab frame:

  constant *effective phase* $\longrightarrow$ *simultaneity hypersurfaces* in lab frame

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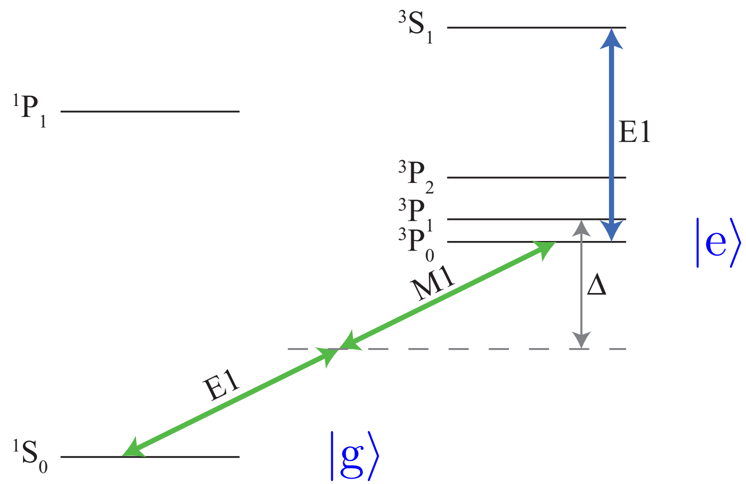

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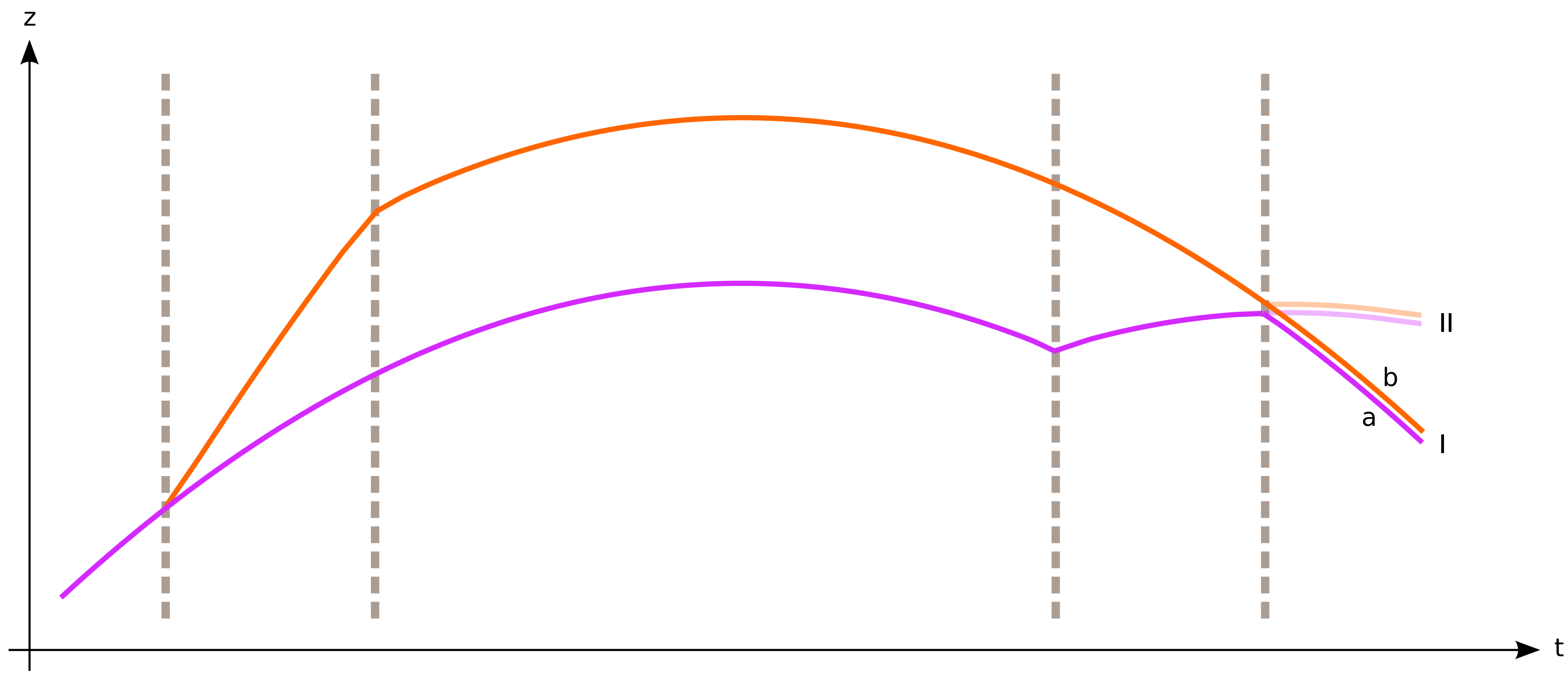
$$e^{-i\omega t} e^{i\mathbf{k}\cdot\mathbf{x}} \times e^{-i\omega t} e^{-i\mathbf{k}\cdot\mathbf{x}} = e^{-i\,2\omega t}$$

Laboratory frame

- Compare *differential phase-shift* measurements for different initialization times:



$$\left(\delta\phi^{(2)}(t'_\mathrm{i}) - \delta\phi^{(1)}(t'_\mathrm{i})\right) - \left(\delta\phi^{(2)}(t_\mathrm{i}) - \delta\phi^{(1)}(t_\mathrm{i})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\, g\, \Delta z\,(t'_\mathrm{i} - t_\mathrm{i})/\hbar$$

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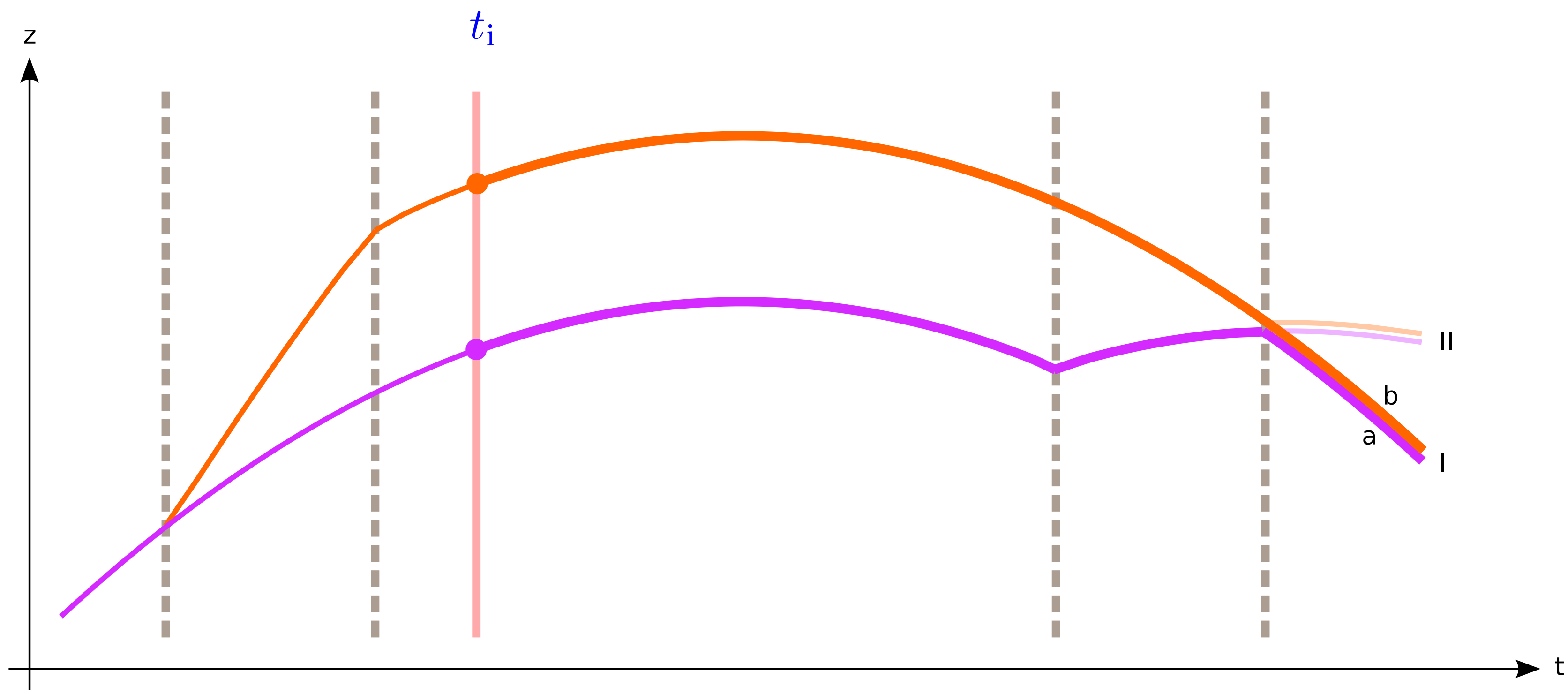

$$\left(\delta\phi^{(2)}(t_\mathrm{i}') - \delta\phi^{(1)}(t_\mathrm{i}')\right) - \left(\delta\phi^{(2)}(t_\mathrm{i}) - \delta\phi^{(1)}(t_\mathrm{i})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\, g\, \Delta z\,(t_\mathrm{i}' - t_\mathrm{i})/\hbar$$

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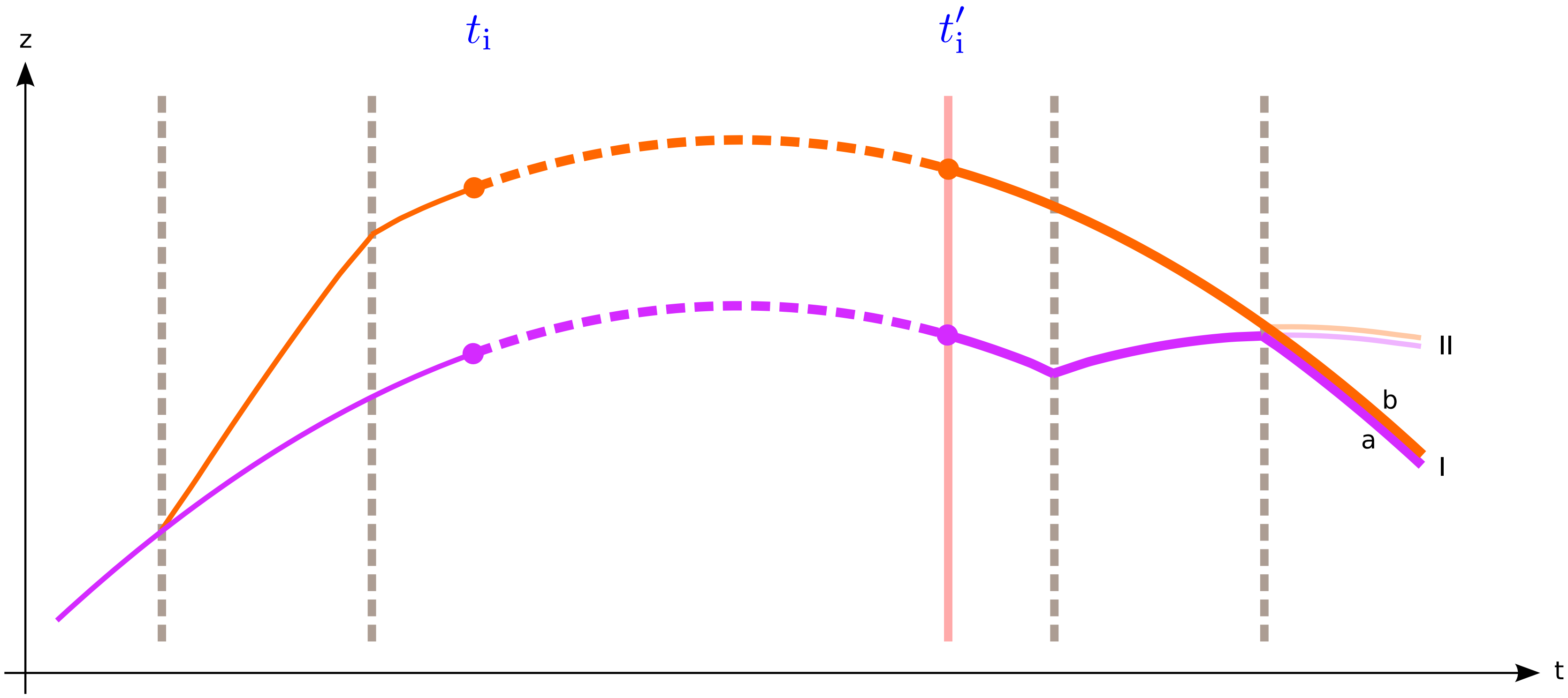

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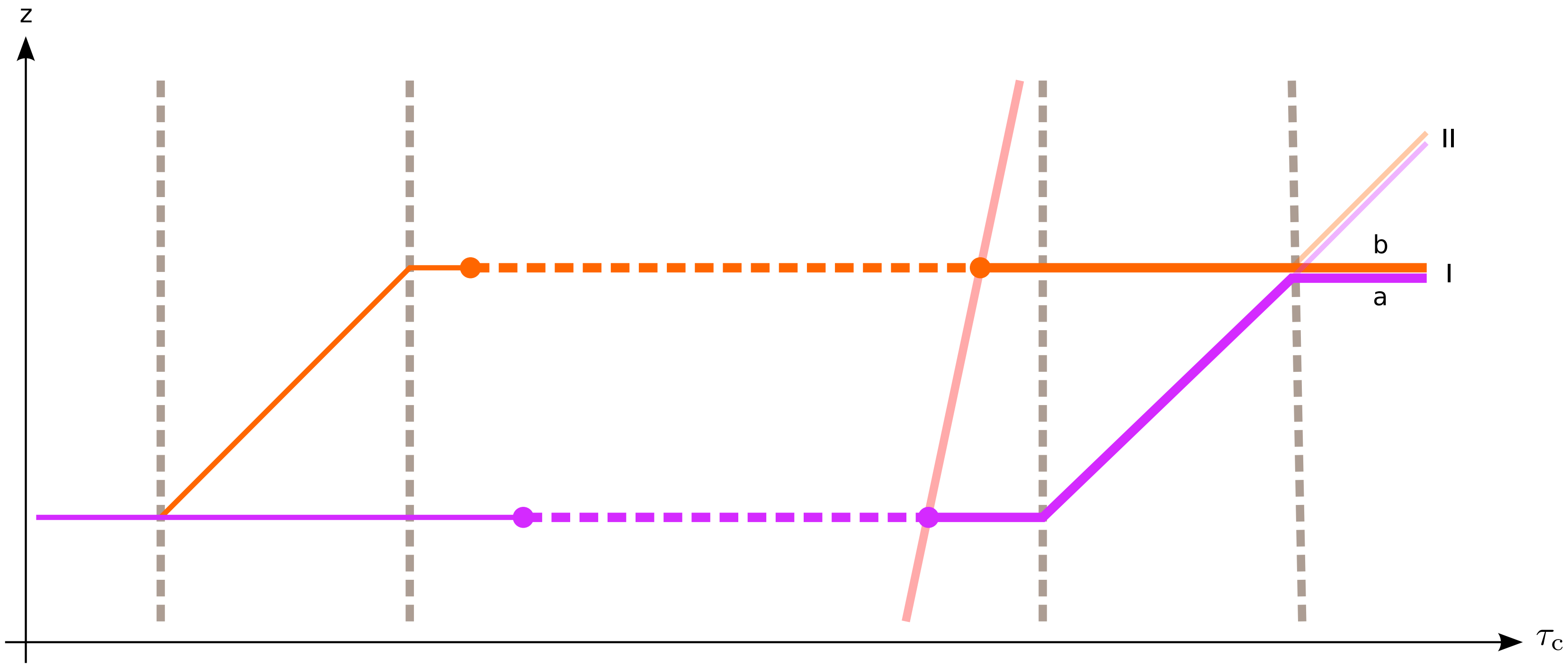
Freely falling frame

- Relativity of simultaneity: $\Delta\tau_{\mathrm{c}} \approx -v(t)\,\Delta z/c^2 = g\,(t - t_{\mathrm{ap}})\,\Delta z/c^2$



$$\left(\delta\phi^{(2)}(t_{\mathrm{i}}') - \delta\phi^{(1)}(t_{\mathrm{i}}')\right) - \left(\delta\phi^{(2)}(t_{\mathrm{i}}) - \delta\phi^{(1)}(t_{\mathrm{i}})\right) = \frac{\Delta E}{2\hbar}\,(\Delta\tau_b - \Delta\tau_a) = \Delta m\,g\,\Delta z\,(t_{\mathrm{i}}' - t_{\mathrm{i}})/\hbar$$

Challenges addressed

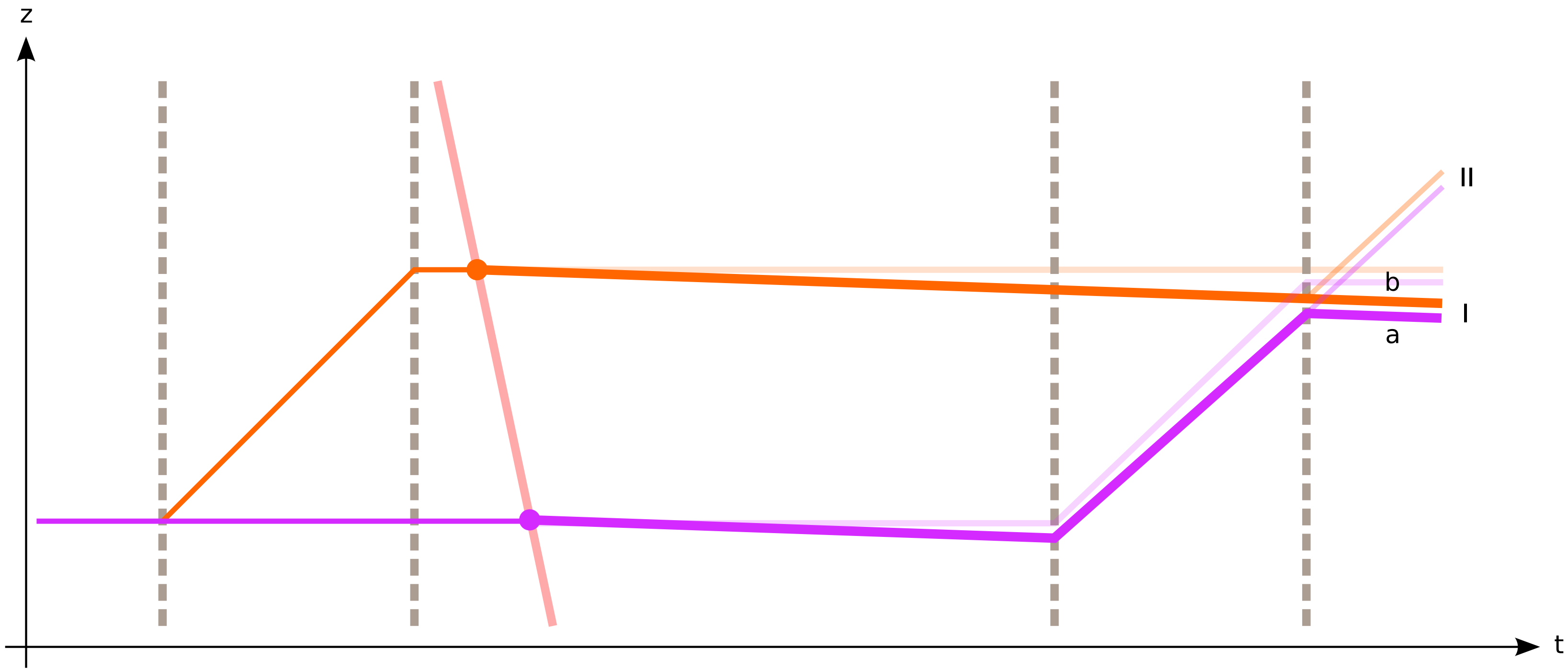
- Differential phase-shift measurement $\longrightarrow$ *precise measurement*, *common-mode rejection* (of *noise* & *systematics*)

- Comparing measurements with different initialization times
  $\longrightarrow$ sensitive to *gravitational redshift* + further *immunity*

- Almost no recoil from *initialization pulse*,
  small *residual recoil* with no impact on gravitational redshift measurement,

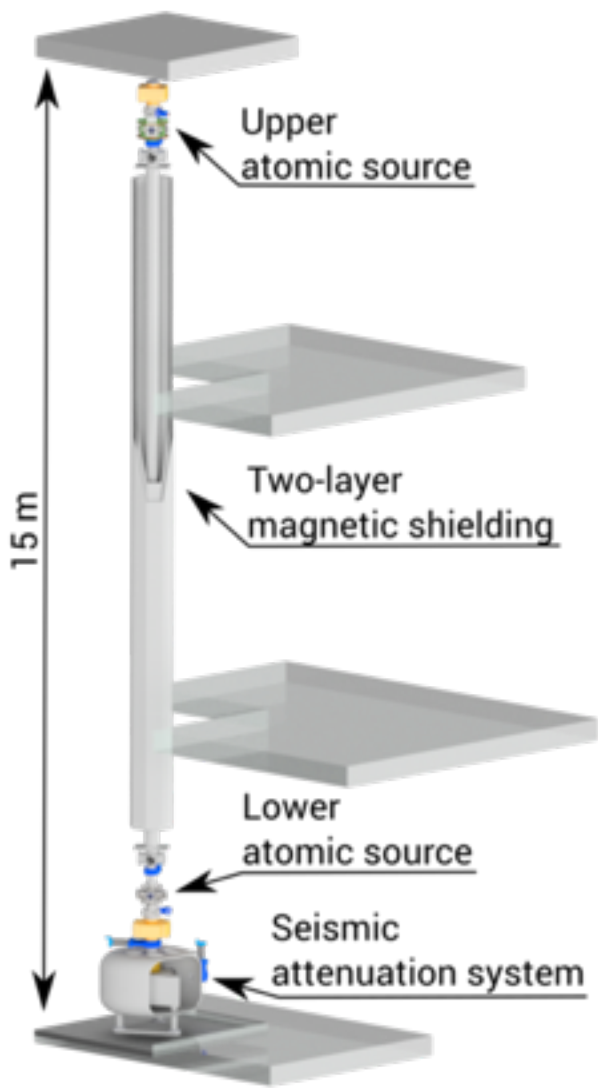
  effect of *differential recoil* from second pair of Bragg pulses cancels out in *doubly differential* measurement.

- Residual recoil with no influence on the *phase-shift* for the *excited* state:



Feasibility and extensions

Feasible implementation



HITec (Hannover)

- 10-m atomic fountains operating with *Sr*, *Yb* in *Stanford* & *Hannover* respectively.
- More than $2\,\mathrm{s}$ of free evolution time.
- *Doubly differential phase shift* of $1\,\mathrm{mrad}$ for

$$\Delta E/\hbar = \omega_0 \approx 2\pi \times 4 \times 10^2\,\mathrm{THz}$$

$$\Delta z = 1\,\mathrm{cm}$$

$$\Delta t_\mathrm{i} = 1\,\mathrm{s}$$

- Resolvable in a *single shot* for atomic clouds with $N = 10^6$ atoms (*shot-noise* limited)
- More compact set-ups possible with *guided* or *hybrid* interferometry (less mature).

Conclusion

- Measurement of relativistic effects in *macroscopically* delocalized quantum superpositions with *quantum-clock interferometry*.

- Important *challenges* in quantum-clock interferometry and its application to gravitational-redshift measurement.

- Promising doubly differential scheme that overcomes them.

- Feasible implementation in facilities soon to become operational.

- Applicable also to more compact set-ups based on *guided* or *hybrid* interferometry.

Acknowledgments

- Collaboration on the experimental realization of the proposed scheme with *Leibniz University Hanover*:

Sina Loriani Dennis Schlippert Ernst Rasel

- Related theoretical work at *Ulm University*:

Proper time in atom interferometers: Diffractive vs. specular mirrors

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Stephan Kleinert Christian Ufrecht

Thank you for your attention.

Gefördert durch:

Bundesministerium
für Wirtschaft
und Energie

aufgrund eines Beschlusses
des Deutschen Bundestages

HORIZON 2020

Q-SENSE
European Union H2020 RISE Project

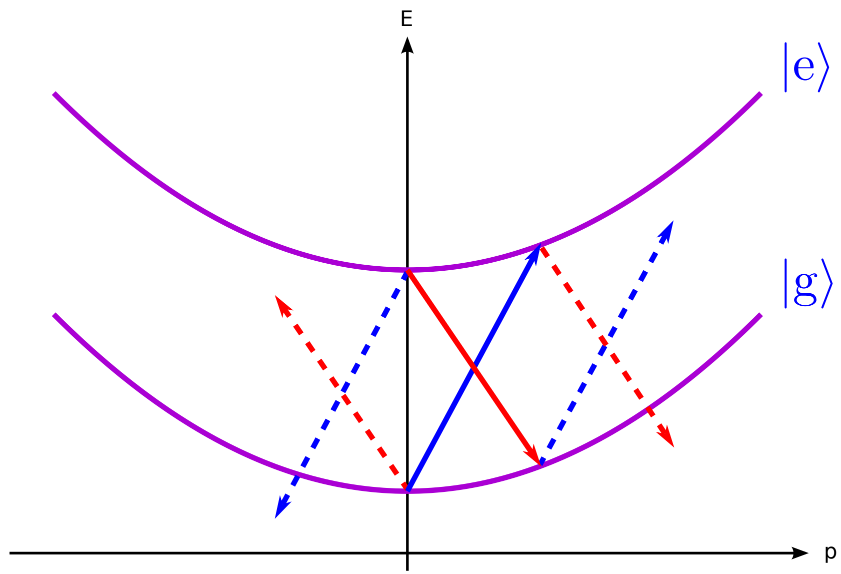
Diffraction of atoms in internal-state superpositions

- Bragg diffraction at the magic wavelength $\longrightarrow$ same Rabi frequency for both internal states.

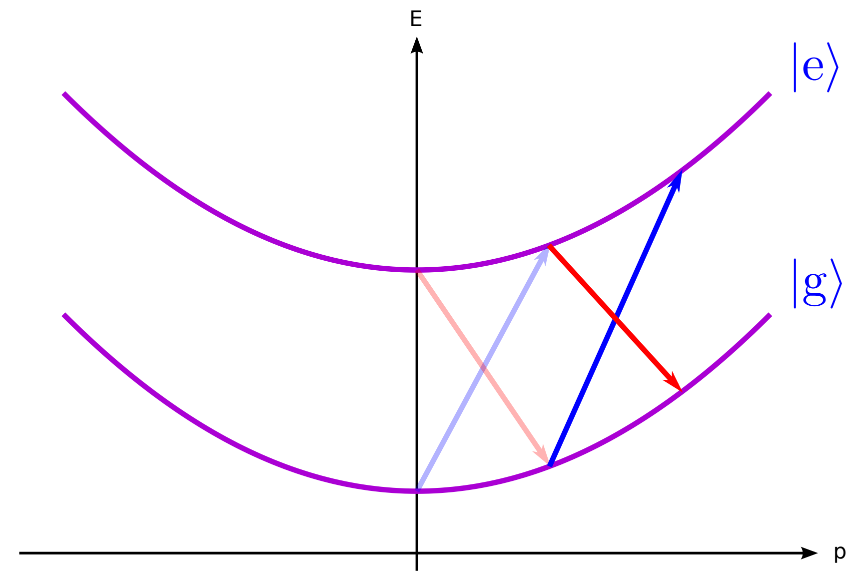
  BUT *high laser power* required due to large detuning.

- Alternative diffraction mechanism based on *simultaneous pair* of single-photon transitions.
  - Applicable to *fermionic* isotopes such as $^{87}\mathrm{Sr}$ and $^{171}\mathrm{Yb}$.
  - Required lasers *already available* in (some of) those facilities.

- Sequence of simultaneous pairs of pulses:



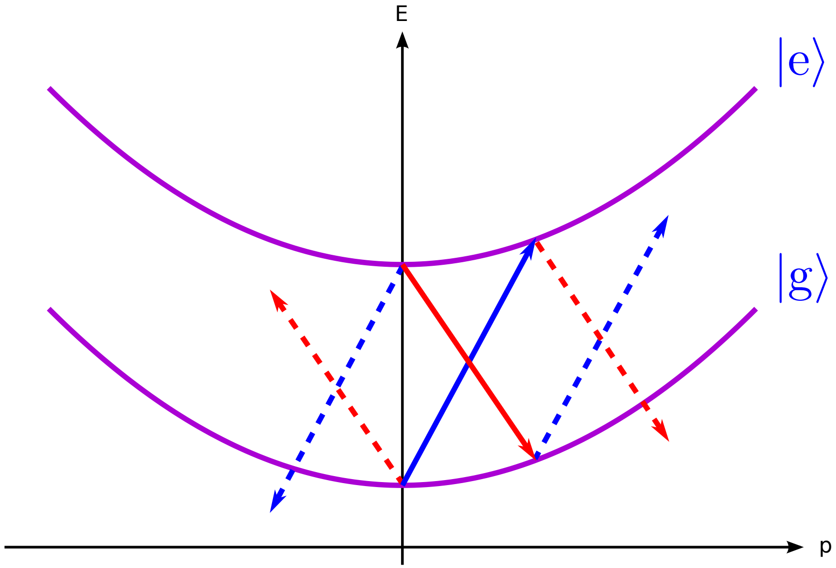
$\pi/2$ pulse



π pulse

- Net result:
  - *internal state* unchanged
  - momentum transfer: *twice single-photon* momentum
  - equal-amplitude superposition: *undiffracted* + *diffracted* wave packet

- Sequence of simultaneous pairs of pulses:



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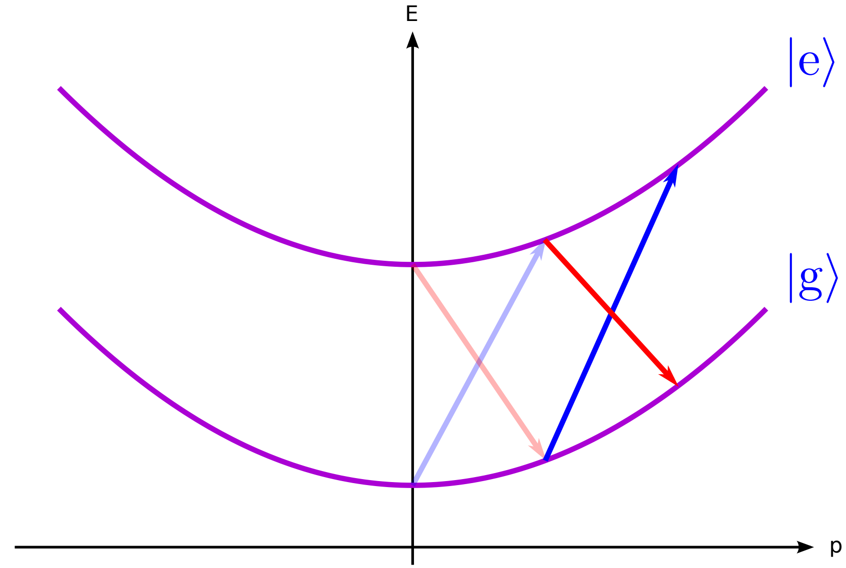


π pulse

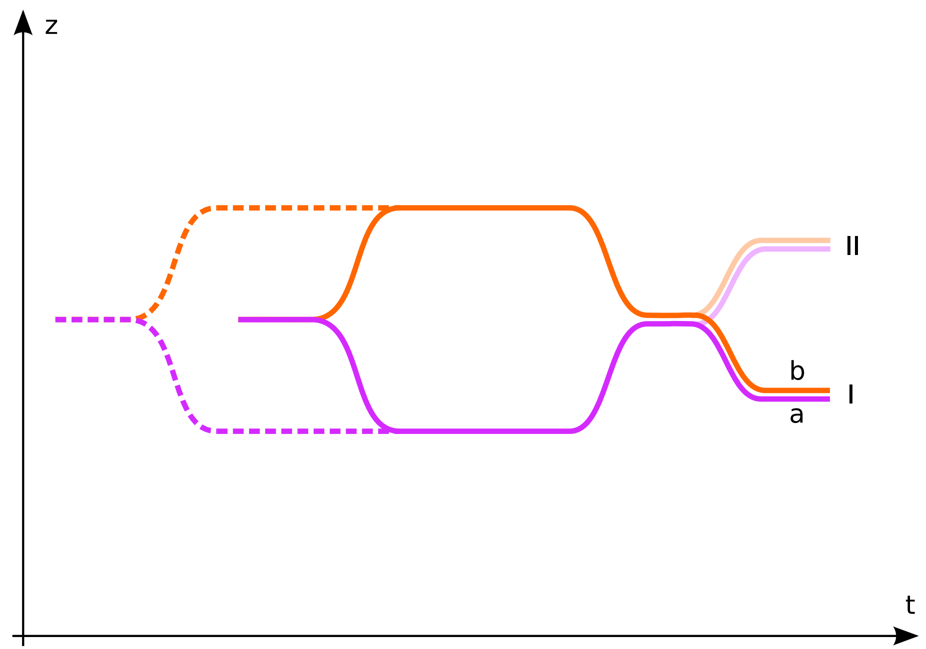
- Same *ac Stark shifts* for both internal states

  → contributions to *differential phase shift* cancel out.

Any *light shifts* cancel out in the *doubly differential measurement* (provided that the *laser intensities* are *stable*).

Extension to guided interferometry

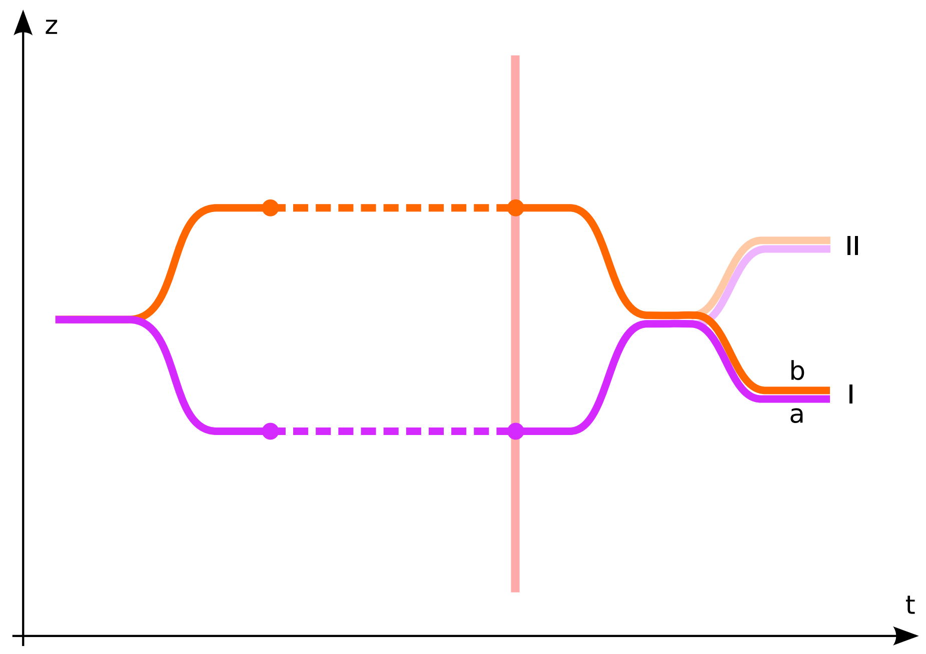
- In principle, guided interferometry can be sensitive to the gravitational redshift:



- Nevertheless, the *doubly differential* measurement scheme has many advantages.

Extension to guided interferometry

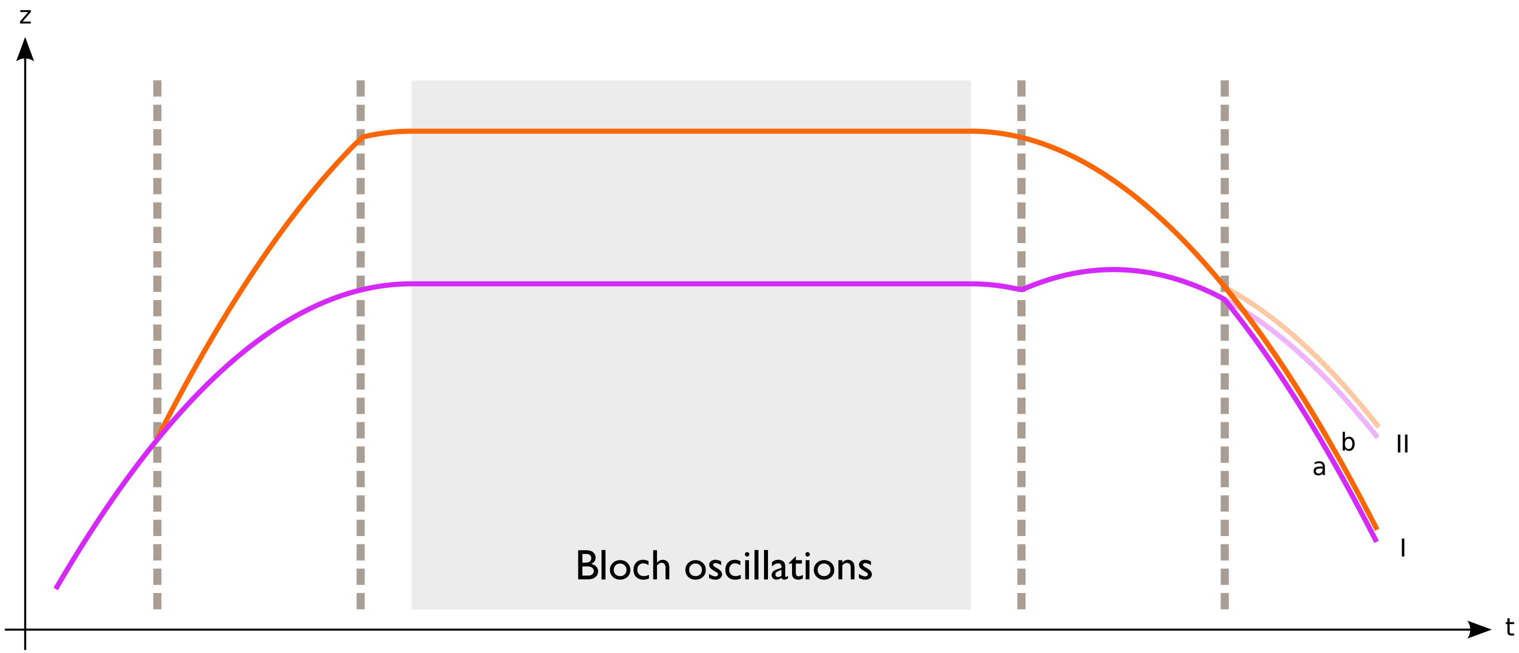
- In principle, guided interferometry can be sensitive to the gravitational redshift:



- Nevertheless, the *doubly differential* measurement scheme has many advantages.

Extension to *hybrid* interferometers

- Intermediate stage with atoms held in an optical lattice, where they undergo Bloch oscillations:



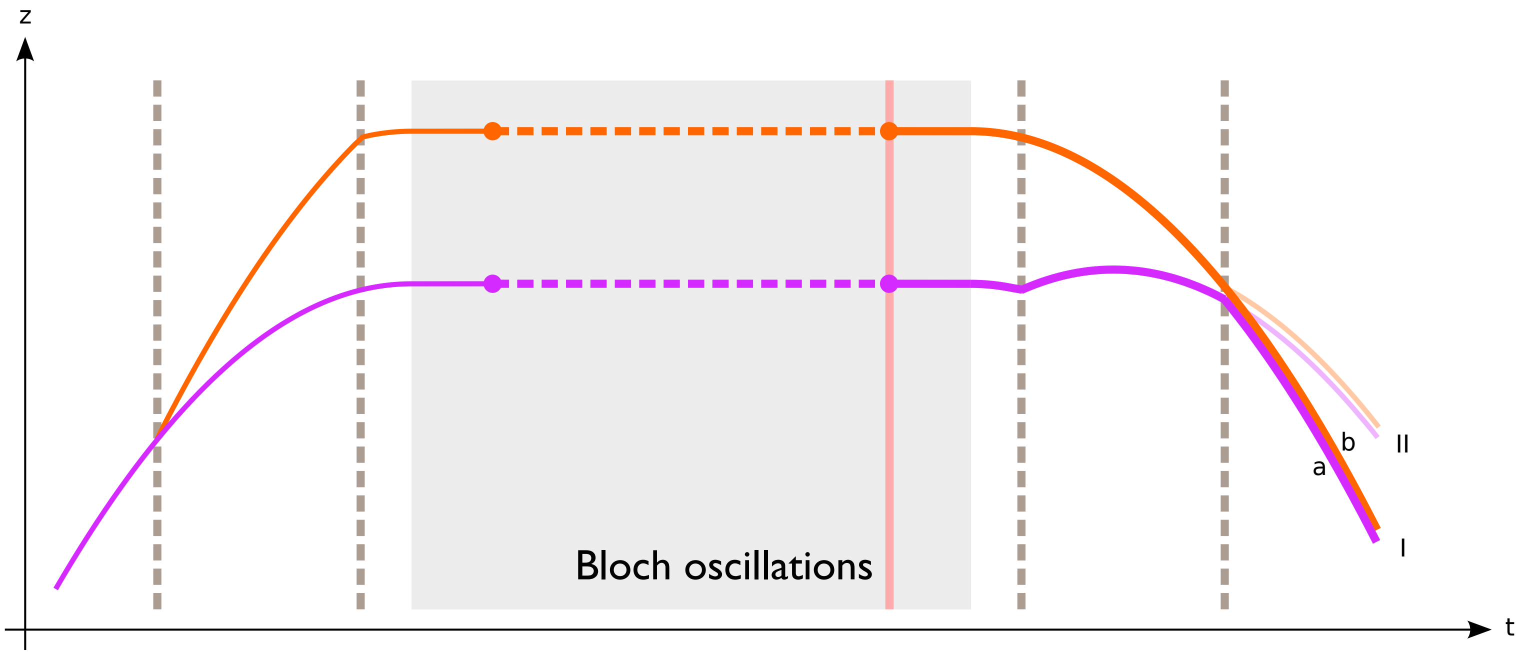
Charriere *et al.*, *Phys. Rev.* A **85** 013639 (2012)

Zhang *et al.*, *Phys. Rev.* A **94** 043608 (2016)

- Similarly to pure *light-pulse* atom interferometers, they are insensitive to the gravitational redshift.

Extension to *hybrid* interferometers

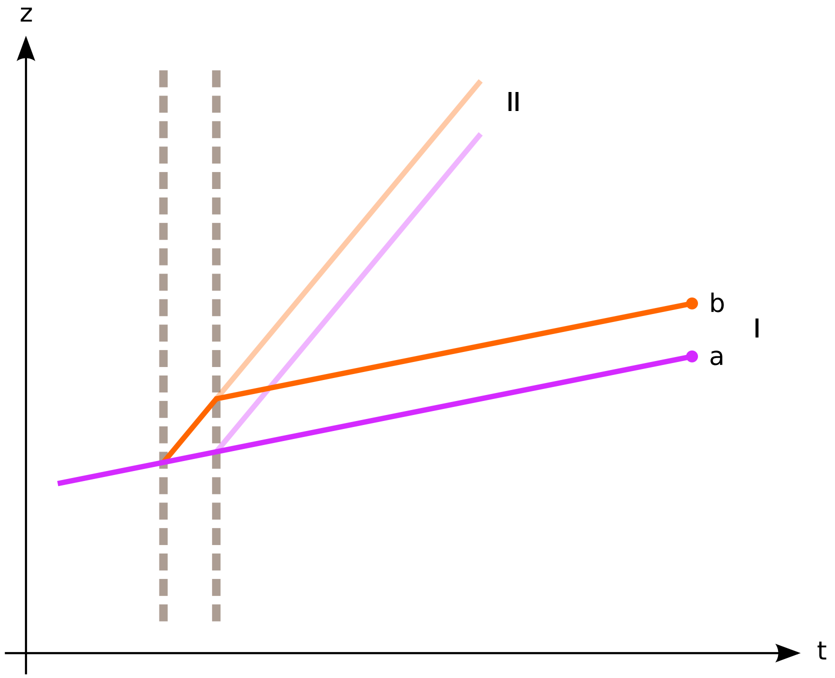
- Intermediate stage with atoms held in an optical lattice, where they undergo Bloch oscillations:

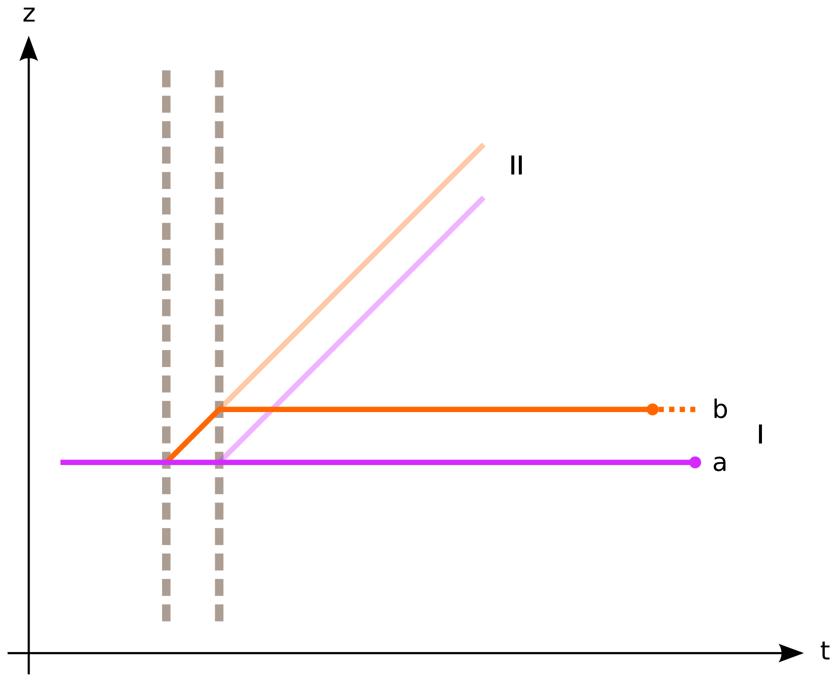


- The *doubly differential* scheme can also be employed for measuring the gravitational redshift.

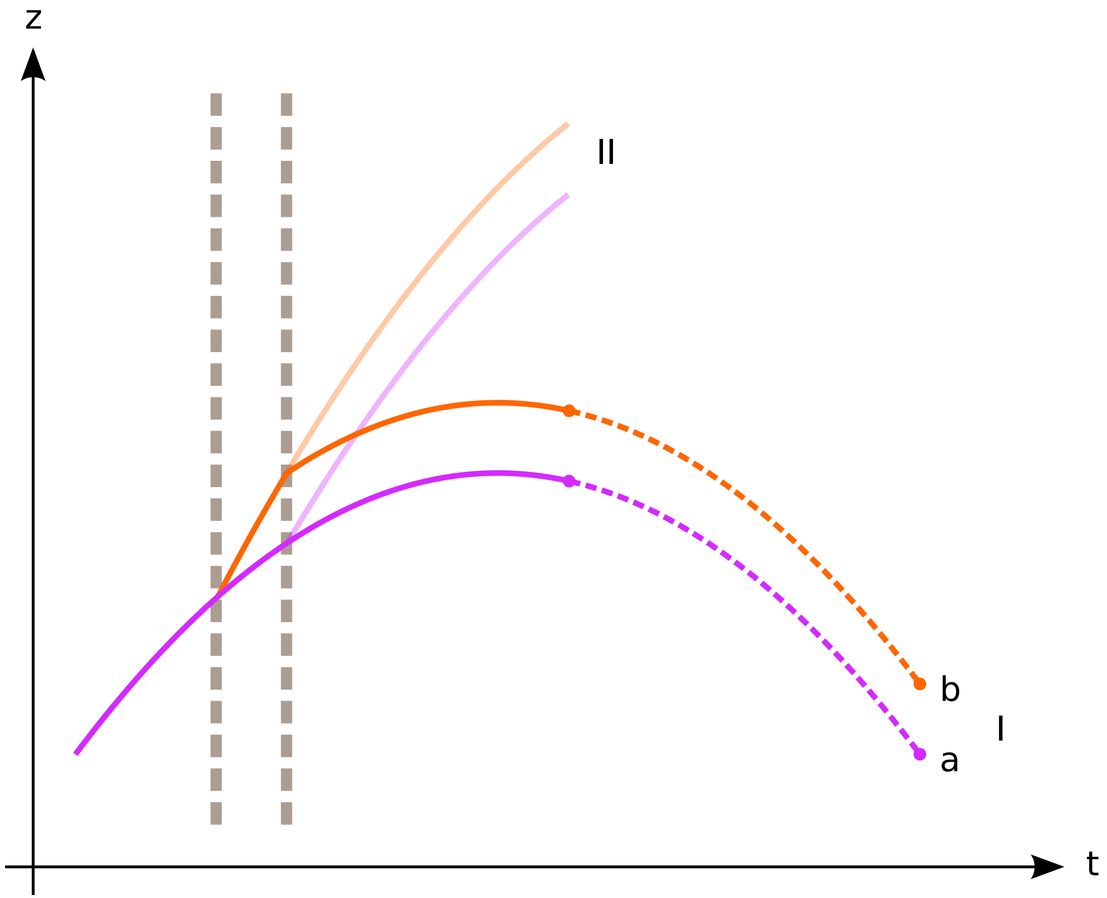
Other aspects

Proper-time difference in open interferometers





Proper-time difference in open interferometers



Proper-time difference and gravity gradients