INSTITUT D'ASTROPHYSIQUE DE PARIS
Unité mixte de recherche 7095 CNRS - Université Pierre et Marie Curie

ESA Topical Teams Workshop

ACES and General Relativity

Munich

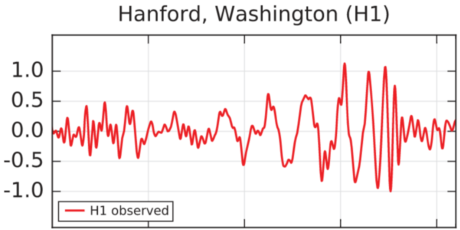
GRAVITATIONAL WAVES & TESTS OF GENERAL RELATIVITY

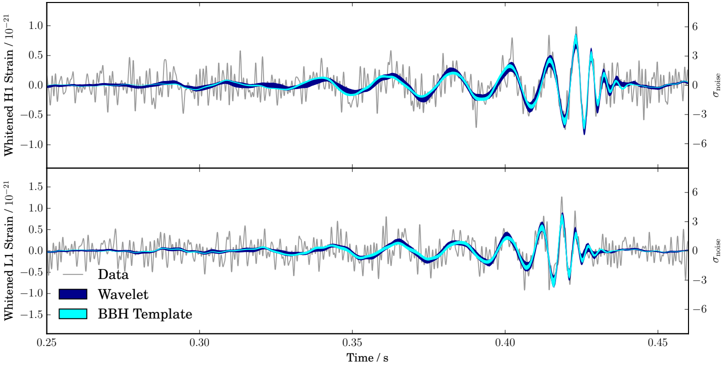
Luc Blanchet

Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
Institut d'Astrophysique de Paris

22 octobre 2018

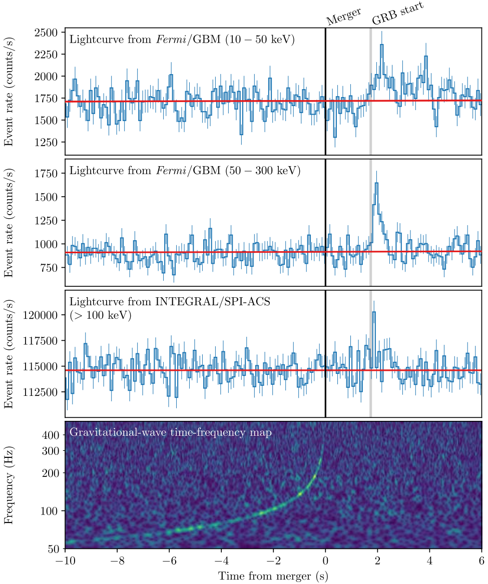
Binary black-hole events [LIGO/Virgo collaboration 2016]





Neutron-star event and multi-messenger astronomy

[LIGO/Virgo collaboration 2017]



The quadrupole formula works! [Einstein 1918; Landau & Lifchitz 1947]

1. The GW frequency is given in terms of the chirp mass $\mathcal{M} = \mu^{3/5} M^{2/5}$ by

$$f = \frac{1}{\pi}\left[\frac{256}{5}\frac{G\mathcal{M}^{5/3}}{c^5}(t_\mathsf{f} - t)\right]^{-3/8}$$

2. Therefore the chirp mass is directly measured as

$$\boxed{\mathcal{M} = \left[\frac{5}{96}\frac{c^5}{G\pi^{8/3}} f^{-11/3}\dot{f}\right]^{3/5}}$$

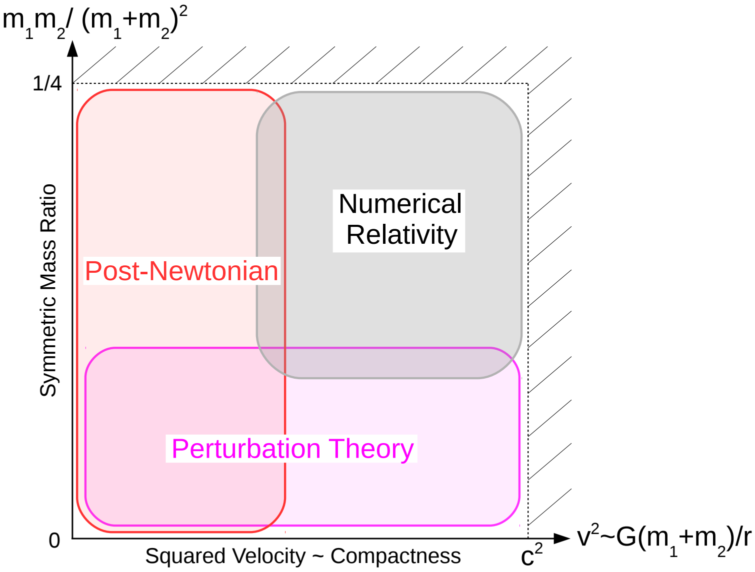
which gives $\mathcal{M} = 30 M_\odot$ thus $M \geqslant 70 M_\odot$

3. The GW amplitude is predicted to be

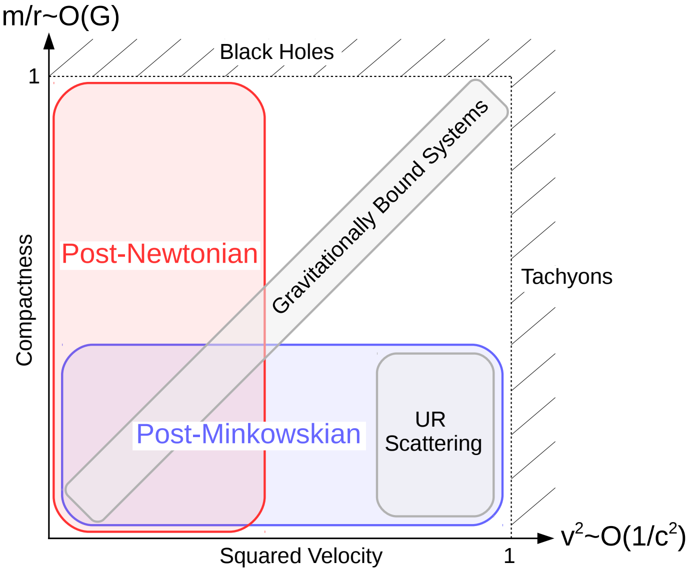
$$h_\mathsf{eff} \sim 4.1\times 10^{-22}\left(\frac{\mathcal{M}}{M_\odot}\right)^{5/6}\left(\frac{100\,\mathsf{Mpc}}{D}\right)\left(\frac{100\,\mathsf{Hz}}{f_\mathsf{merger}}\right)^{-1/6} \sim 1.6\times 10^{-21}$$

4. The distance $D = 400\,\mathsf{Mpc}$ is measured from the signal itself

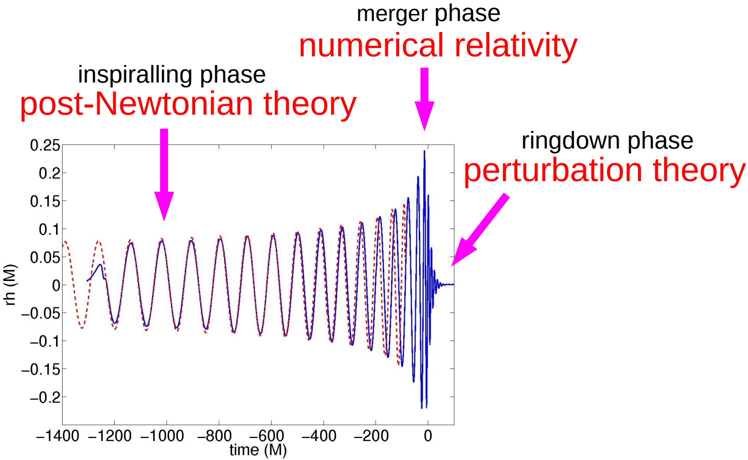
Methods to compute GW templates



Methods to compute GW templates

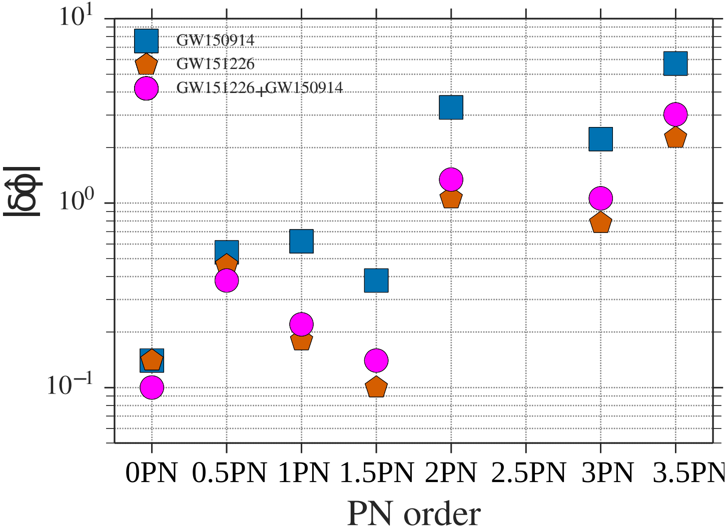


The gravitational chirp of compact binaries

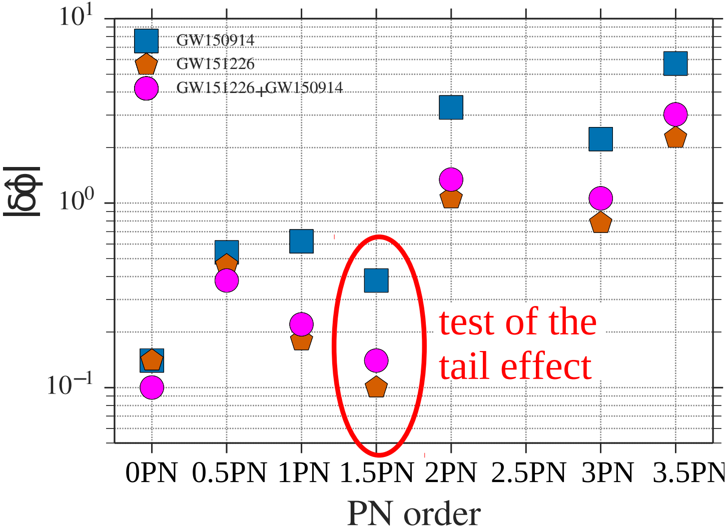


Effective methods such as EOB and IMR that interpolate between the PN and NR are also very important notably for the data analysis of black hole binaries

Measurement of PN parameters [LIGO/VIRGO 2016]

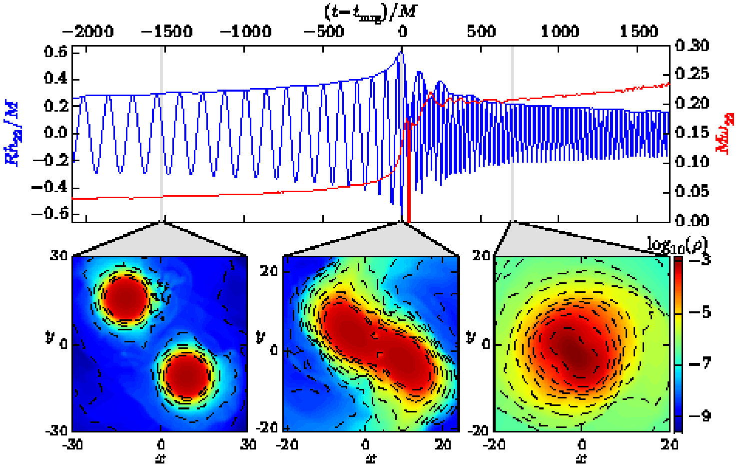


Measuring GW tails [Blanchet & Sathyaprakash 1994, 1995]



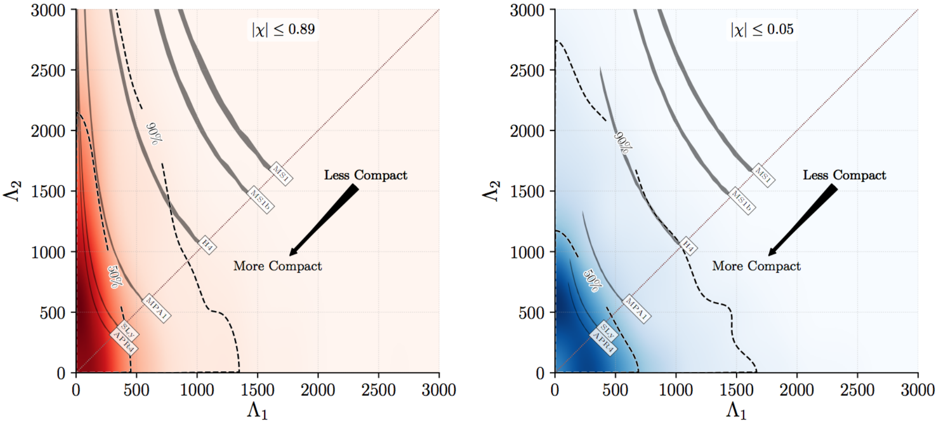
Post-merger waveform of neutron star binaries

[Shibata *et al.*; Rezzolla *et al.* 1990-2010s]



Constraining the neutron star equation of state

[LIGO/Virgo 2017]



$$\Lambda = \frac{2}{3} k_2 \left(\frac{c^2 a}{G m} \right)^5$$

Black holes have no hair

1. Exterior geometry of the rotating BH solution [Kerr 1963]

$$
\begin{aligned}
g_{00} &= -1 + \frac{M}{r} + \frac{M_2 P_2(\cos\theta)}{r^3} + \frac{M_4 P_4(\cos\theta)}{r^5} + \cdots \\
g_{0\varphi} &= \frac{J}{r^2} + \frac{J_3 \tilde{P}_3(\cos\theta)}{r^4} + \frac{J_5 \tilde{P}_5(\cos\theta)}{r^6} + \cdots
\end{aligned}
$$

2. The no hair theorem states (with $M_0 = M$, $J_1 = J$, $a = J/M$) [Hansen 1974]

$$\boxed{M_\ell + \mathrm{i} J_\ell = M \, (\mathrm{i}a)^\ell}$$

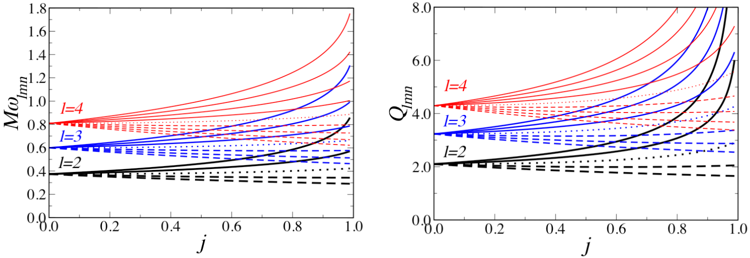
3. The quadrupole moment is determined from the mass and spin

$$\boxed{M_2 = -M\, a^2 = -\frac{J^2}{M}}$$

Counting BH hairs with the ringdown radiation

- The merger of two black holes produces a distorted BH who emits ringdown radiation to shed hair
- The frequency modes of ringdown radiation [*e.g.* Berti, Cardoso & Will 2006]

$$\omega = \omega_{\ell mn}\left[1 + \frac{\mathrm{i}\pi}{2Q_{\ell mn}}\right]$$



$$j = \frac{J}{M^2} = \frac{a}{M}$$

Gravitational echoes [see e.g. Cardoso & Pani 2017]

- Suppose that the object formed by the merger of two BHs contains a material surface between the horizon at $2M$ and the photon sphere at $3M$,

$$R = 2M\,(1+\epsilon) \quad \text{with} \quad \epsilon \ll 1$$

- For instance the surface could be at the Planck length from the horizon and be related to quantum effects occuring at the horizon scale, in that case

$$\epsilon_{\text{Planck}} = 10^{-38}\left(\frac{M_\odot}{M}\right)$$

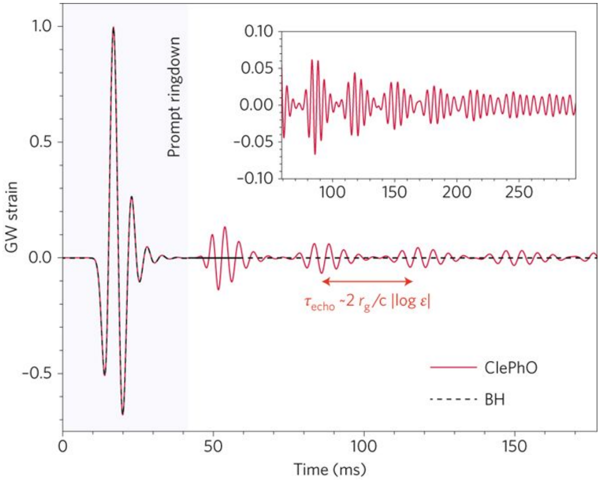
- The ringdown radiation is followed by a series of echoes separated by

$$\boxed{\tau_{\text{echo}} \sim 2M\big|\ln\epsilon\big|}$$

- Echos may also reveal exotic compact objects (ECO) with radius

$$\underbrace{\tfrac{9}{4}M}_{\text{Buchdahl limit}} \leqslant R < 3M$$

Gravitational echoes [see e.g. Cardoso & Pani 2017]



$$f_{\text{ringdown}} \sim 10^3\,\text{Hz}\left(\frac{100M_\odot}{M}\right) \qquad \tau_{\text{echo}}^{\text{Planck}} \sim 10^{-2}\,\text{s}\left(\frac{M}{100M_\odot}\right)$$

GW solutions in metric theories of gravity

1. Small perturbation of the metric around flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad |h_{\mu\nu}| \ll 1$$

2. Restrict attention to theories admitting GW solutions propagating at the speed of light: $c_g = 1$. Far from the sources the waves are planar, hence

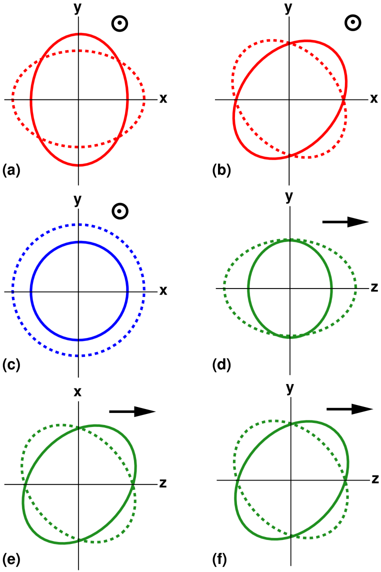
$$\Box h_{\mu\nu} = 0 \quad \Longleftrightarrow \quad h_{\mu\nu} = h_{\mu\nu}(t-z)$$

3. From the linearized Bianchi's identity obtain

$$\boxed{\Box R_{\mu\nu\rho\sigma} = 0 \quad \Longleftrightarrow \quad R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma}(t-z)}$$

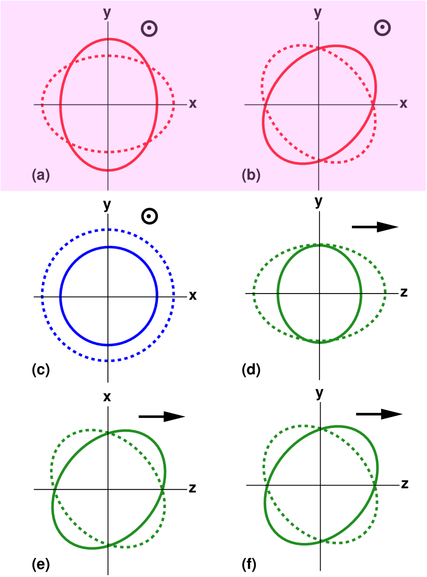
   showing that GWs have an invariant, coordinate-independent meaning
4. The six components R_{0i0j} (where $i, j = x, y, z$) represent six independent components (polarization modes)
5. In GR $R_{\mu\nu} = 0$ hence there are only two independent polarization modes

GW polarization modes in metric theories of gravity



- General Relativity
- Scalar-Tensor theory [*e.g.* Will 1993]
- Massive Gravity theory [*e.g.* de Rham 2014]
- Scalar-Vector-Tensor [Sagi 2010]

GW polarization modes in metric theories of gravity



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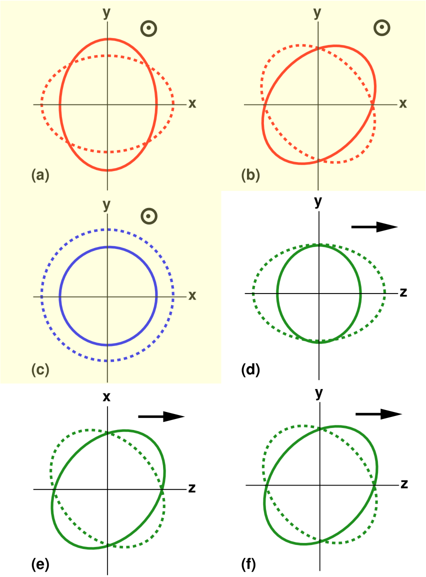

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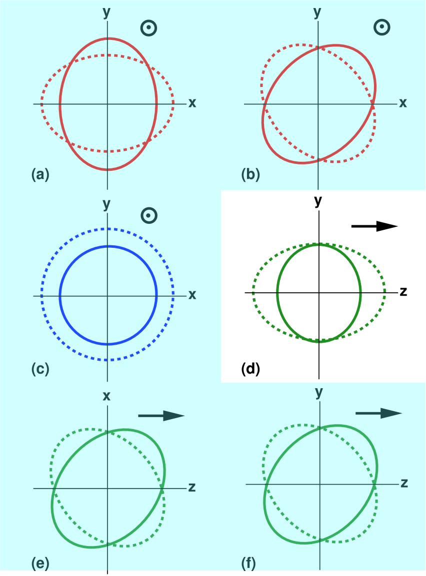

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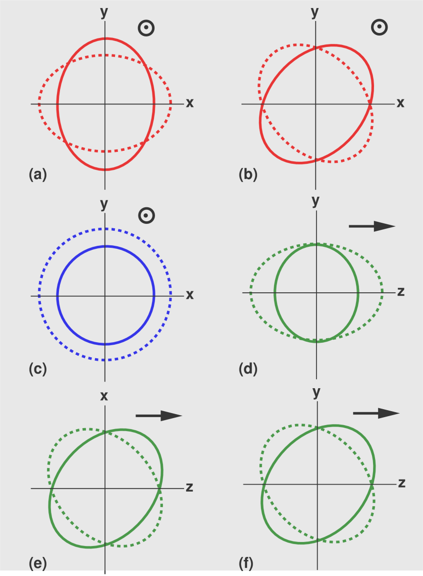

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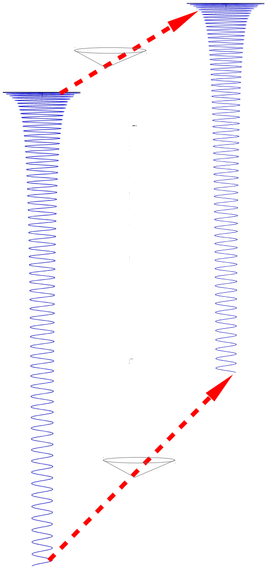
Bounding the mass of the graviton [Will 1998]



- Dispersion relation for a massive graviton

$$\boxed{\frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E_g^2}} \quad \text{with} \quad E_g = \hbar\omega_g$$

- The frequency of GW sweeps from low to high frequency during the inspiral and the speed of GW varies from lower to higher (close to c) speed at the end
- The constraint is [LIGO/Virgo 2016]

$$\boxed{m_g \lesssim 10^{-22}\,\text{eV}} \quad \Leftrightarrow \quad \lambda_g \gtrsim 0.02\ \text{ly}$$

Nonlinear ghost-free massive (bi-)gravity

[de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

- The theory (called dRGT) is defined non-perturbatively as

$$\boxed{S = \int \mathrm{d}^4 x \left[\frac{M_g^2}{2} \sqrt{-g} R_g + \overbrace{m_\mathrm{g}^2 \sqrt{-g} \sum_{n=0}^{4} \alpha_n e_n(X)}^{\text{ghost-free interaction mass term}} + \frac{M_f^2}{2} \sqrt{-f} R_f \right]}$$

where the interaction between the two metrics is defined from the elementary symmetric polynomials $e_n(X)$ of the square root matrix $X = \sqrt{g^{-1} f}$

- Such massive gravity theories modify GR at large cosmological distances and are motivated by the problem of the cosmological constant

$$\boxed{\Lambda \sim \frac{m_\mathrm{g}^2 c^2}{\hbar^2}} \quad \Rightarrow \quad m_\mathrm{g} \sim 3.7\, 10^{-33}\, \text{eV}$$

Test of the strong equivalence principle

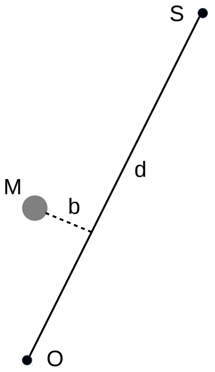
[see e.g. Desai & Kahya 2016]

1. Cumulative Shapiro time delay due to the gravitational potential of the dark matter distribution
2. Violation of the EP is quantified by a PPN like parameter γ_a with $a = \mathrm{GW}, \mathrm{EM}$. For a spherical mass distribution

$$\Delta t^a_{\text{Shapiro}} = (1+\gamma_a)\frac{GM}{c^3}\ln\left(\frac{d}{b}\right)$$

3. Main contributions are from the host galaxy NGC4993 and the Milky Way ($M_{\text{MW}} = 5.6\,10^{11}\,M_\odot$). Assuming an isothermal density profile for DM the GR delay is $400\,$days
4. The observed difference in arrival time $\Delta t = 1.7\,$s yields

$$\boxed{|\gamma_{\text{GW}} - \gamma_{\text{EM}}| \lesssim 10^{-7}}$$



Generalized scalar-tensor theories

1. Traditional scalar-tensor theories [Jordan 1949; Fierz 1956; Brans & Dicke 1961]

$$L\big[\phi, \nabla_\mu\phi\big] = F(\phi)R - Z(\phi)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - U(\phi) + \underbrace{L_\mathsf{m}\big[\psi_\mathsf{m}, g_{\mu\nu}\big]}_{\text{universal coupling to } g_{\mu\nu}}$$

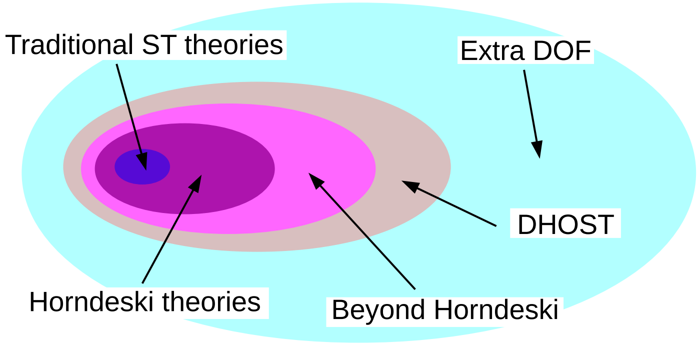
2. Generalized theories with second-order derivatives

$$L\big[\phi, \nabla_\mu\phi, \nabla_\mu\nabla_\nu\phi\big]$$

generically contain an extra scalar degree of freedom and lead to instabilities (in particular, the Hamiltonian is not bounded from below) [Ostrogradsky 1850]

3. It is however possible to avoid the instabilities for special choices of second-order Lagrangians

Generalized scalar-tensor theories



- Horndeski theories [Horndeski 1974]
- Beyond Horndeski [Gleyzes, Langlois, Piazza & Vernizzi 2014]
- Degenerate Higher Order Scalar Tensor (DHOST) [Langlois & Noui 2016]

Case of Horndeski theories [Horndeski 1974; rediscoverd Charmousis *et al.* 2012]

1. Most general theory with at most second-order Euler-Lagrange equations
2. It involves four functions of ϕ and the kinetic term $X = \nabla_\mu\phi\nabla^\mu\phi$

$$\begin{aligned} L = {} & G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R \\ & - 2G_{4,X}(\phi, X)\left(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right) + G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} \\ & + \frac{1}{3}G_{5,X}(\phi, X)\Big(\Box\phi^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu\nu}\phi_{\mu\rho}\phi^{\rho}_{\nu}\Big) \end{aligned}$$

3. Perturbations around a cosmological background $a(t)$, $\phi(t)$ in ADM form

$$L_{\text{quad}} = a^3\frac{M^2}{2}\left[\delta K_{ij}\delta K^{ij} - \Big(1 + \frac{2}{3}\alpha_L\Big)\delta K^2 + (1 + \alpha_T)R\frac{\delta\sqrt{\gamma}}{a^3} + \cdots\right]$$

4. Lagrangian for the tensor modes

$$L^T_{\text{quad}} = a^3\frac{M^2}{8}\left[\delta\dot{\gamma}_{ij}^2 - (1 + \alpha_T)\frac{(\partial_k\delta\gamma_{ij})^2}{a^2}\right]$$

Dark energy after GW170817 [Bettoni *et al.* 2017; Creminelli & Vernizzi 2017]

1. The observed time delay between GW170817 and the GRB constrains

$$|c_\text{g} - c_\text{em}| \lesssim 10^{-15} c$$

2. In Horndeski theory the speed of gravitational waves is

$$c_\text{g} = c_T = \sqrt{1 + \alpha_T}$$

3. Imposing the speed of GWs to be one ($\alpha_T = 0$) drastically reduces the space of allowed theories (with $B_4 = G_4 + \frac{X}{2} G_{5,\phi}$)

$$\boxed{L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi)R}$$

4. The third term simply recovers the standard conformal coupling of ST theories, and the second one is a Galileon type term still able to produce the accelerated expansion

Tensor-vector-scalar theory (TeVeS) [Bekenstein 2004; Sanders 2005]

$$L_{\text{scalar-tensor}} = \frac{1}{16\pi}\left[R + 2a_0^2\, F\left((g^{\mu\nu} - A^\mu A^\nu)\frac{\partial_\mu\phi\partial_\nu\phi}{a_0^2}\right)\right] + L_{\text{m}}[\Psi_{\text{m}}, \tilde{g}_{\mu\nu}]$$

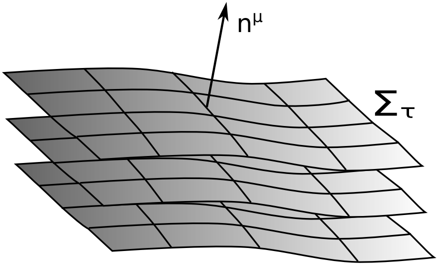
$$L_{\text{vector}} = \frac{1}{16\pi}\left[k\, F^{\mu\nu}F_{\mu\nu} + \underbrace{\lambda\big(A^\mu A_\mu + 1\big)}_{\text{Lagrange constraint}}\right]$$

- The theory is motivated by reproducing the phenomenology of MOND at galactic scales without dark matter and F is related to the MOND function
- Gravitational waves couple to the Einstein-frame metric $g_{\mu\nu}$ produced by GR without dark matter
- Ordinary matter couples to the Jordan-frame metric $\tilde{g}_{\mu\nu}$ which is a disformally transformed metric that would be produced by GR with dark matter

$$\boxed{\tilde{g}_{\mu\nu} = e^{2\phi}(g_{\mu\nu} + A_\mu A_\nu) - e^{-2\phi}U_\mu U_\nu}$$

- TeVeS and other dark matter emulators like MOG [Moffat 2006] are excluded by GW170817 [Boran, Desai, Kahya & Woodard 2018; Wang *et al.* 2018]

Khronometric theory [Blanchet & Marsat 2011; Sanders 2011]



To get rid of the Lagrange constraint we choose n_μ to be hypersurface orthogonal

$$n_\mu = -N\partial_\mu \tau$$

where τ is a dynamical scalar field called the Khronon and where

$$N = \frac{1}{\sqrt{-g^{\rho\sigma}\partial_\rho \tau \partial_\sigma \tau}}$$

Looking for a modification of GR for weak accelerations we can take the acceleration of the congruence of worldlines orthogonal to the foliation

$$a_\mu = n^\nu \nabla_\nu n_\mu = D_\mu \ln N$$

Khronometric theory [Blanchet & Marsat 2011; Sanders 2011]

Like for Einstein-Æther the theory admits a covariant formulation

$$\mathcal{L}_{\text{Einstein-Khronon}} = \frac{\sqrt{-g}}{16\pi}\Big[R - 2F(a)\Big] + \mathcal{L}_{\text{m}}[g_{\mu\nu}, \psi_{\text{m}}]$$

but its content is best understood in adapted coordinates where $t = \tau$

$$\mathcal{L}_{\text{Einstein-Khronon}} = \frac{\sqrt{\gamma}}{16\pi}N\Big[\mathcal{R} + K_{ij}K^{ij} - K^2 - 2F(a)\Big] + \mathcal{L}_{\text{m}}[N, N_i, \gamma_{ij}, \psi_{\text{m}}]$$

where $a_\mu = D_\mu \ln N$ and N, N_i, γ_{ij} are purely geometrical quantities

- For a choice of function F we recover GR+Λ in the strong-field regime $a \gg a_0$ and MOND in the weak-field regime $a \ll a_0$
- The theory is still viable with respect to GW170817
- But unfortunately, no viable cosmology

Limit on the number of space-time dimensions

[Parda, Fishbach, Holz & Spergel 2018]

- Extra-dimensional theories of gravity generically predict a deviation from the law $h^{\mathsf{GW}} \propto 1/d_L$ due to gravitational leakage into extra dimensions

$$h^{\mathsf{GW}} \propto \frac{1}{d_L^{\frac{D-2}{2}}}$$

- For instance theories that admit a screening scale R_c behave like GR below this scale but exhibit gravitational leakage above (where n gives the transition steepness) [Deffayet & Menou 2007]

$$h^{\mathsf{GW}} \propto \frac{1}{d_L\left(1+\left(\frac{d_L}{R_\mathsf{c}}\right)^{\frac{n(D-4)}{2}}\right)^{1/n}}$$

- The measurement of GW170817 shows that $D = 3.98^{+0.07}_{-0.09}$ using the Planck value for the Hubble-Lemaître constant