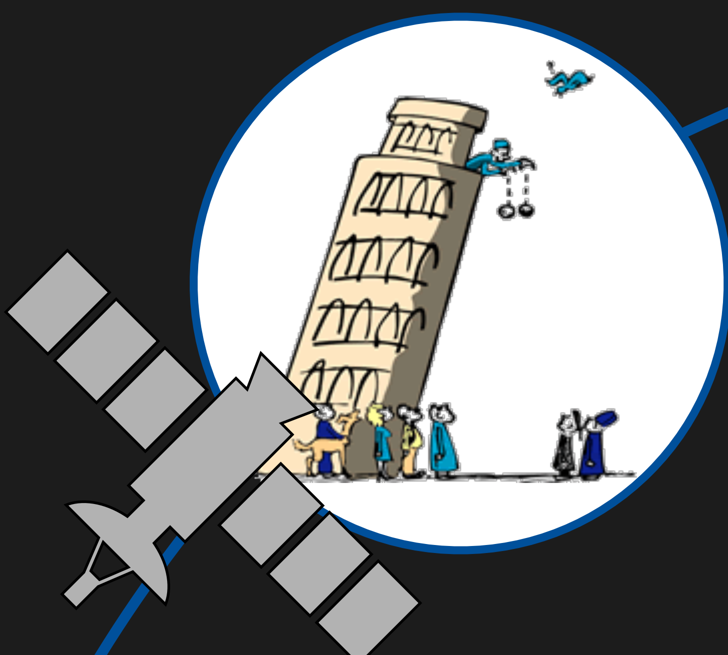
Gravity gradient cancellation in satellite quantum tests of the Equivalence Principle



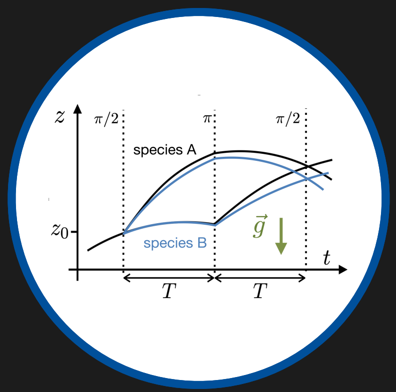
Leibniz Universität Hannover

l'Observatoire de Paris — SYRTE

Systèmes de Référence Temps-Espace

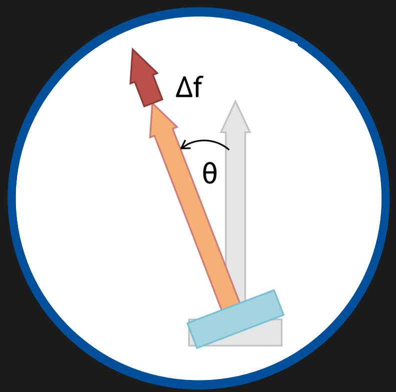
Outline

I - The Einstein Equivalence Principle

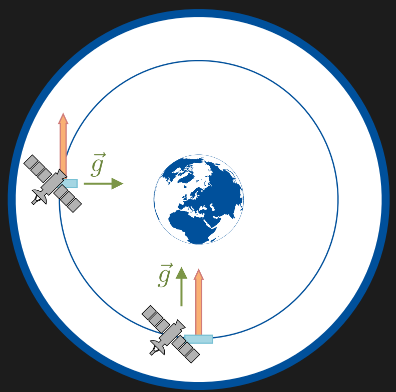


II - Atom Interferometry

III - Co-location problem



IV - Gradient cancellation



V - Signal Demodulation

I - The Einstein Equivalence Principle

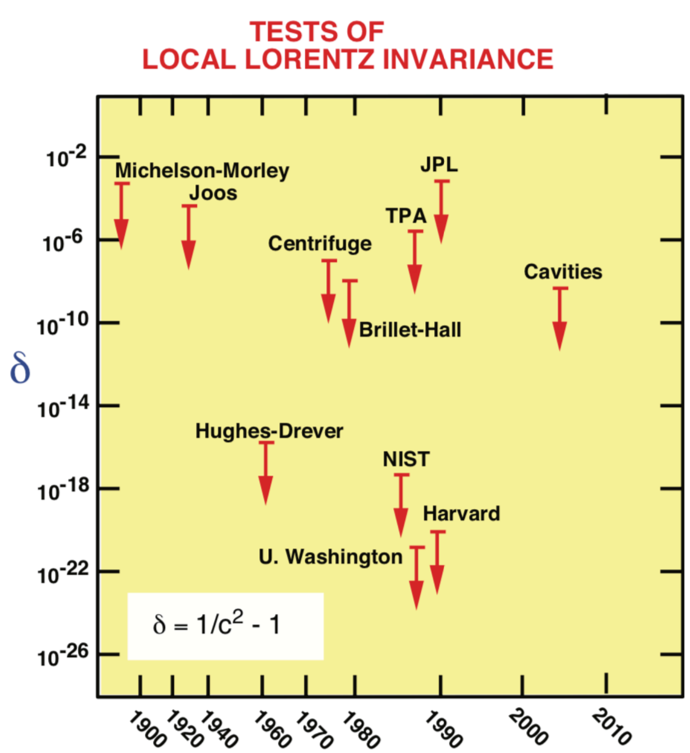
Implies metric theory of gravity. Modern formulation:

1 - LLI: independence of reference frame velocity

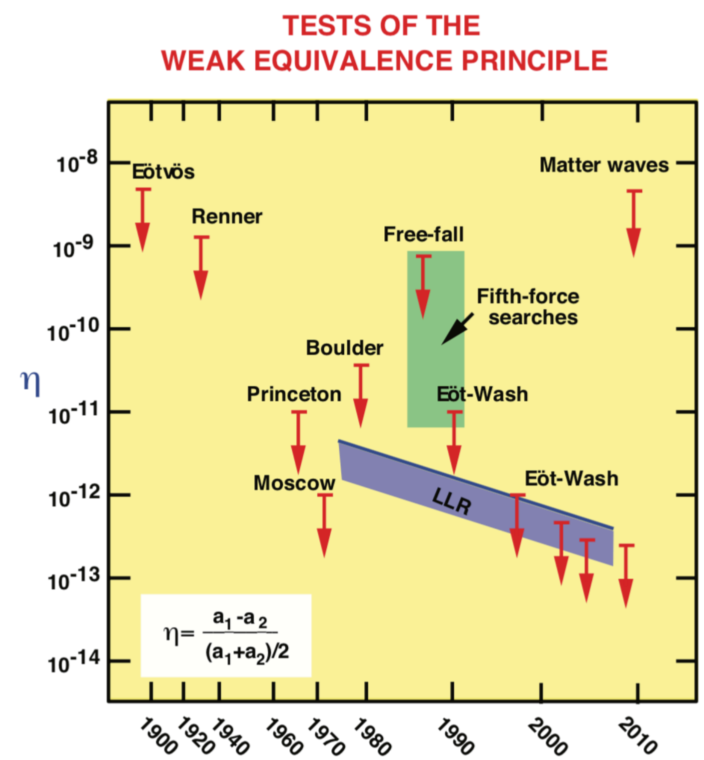
2 - UFF/WEP: free-fall trajectory independent of internal composition

3 - LPI/UGR: independence of location

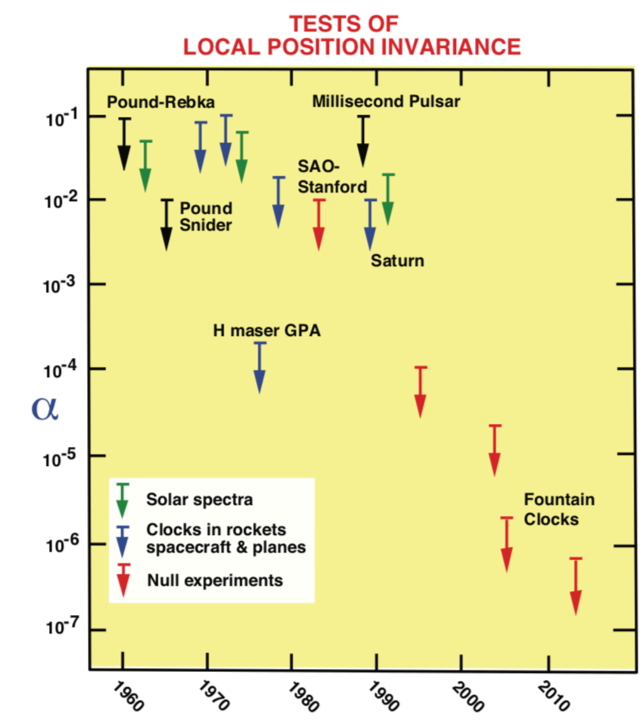
Will: Living Reviews in Relativity **17**(1) (2014)



$$\delta = \frac{1}{c^2} - 1$$



$$\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2}$$

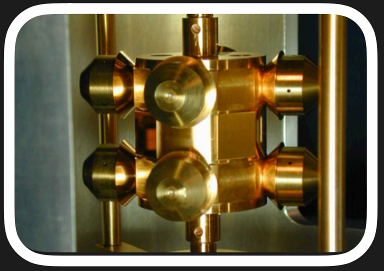


$$Z = (1 + \alpha)\frac{\Delta U}{c^2}$$

I - The Einstein Equivalence Principle

Weak Equivalence Principle: Free fall trajectory independent of internal composition

(Classical) Tests:

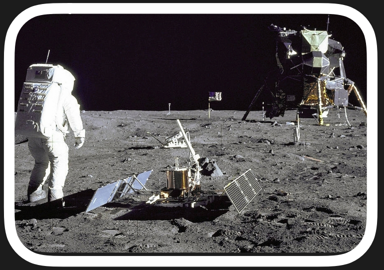


©Wagner et al.

Torsion balance

Wagner et al., CQG **29**, 184002 (2012)

$$\eta(\text{Be,Ti}) = (0.3 \pm 1.8) \times 10^{-13}$$
$$\eta(\text{Be,Al}) = (-0.7 \pm 1.3) \times 10^{-13}$$

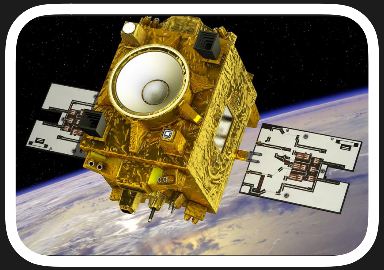


©NASA

Lunar laser ranging

Hofmann et Müller, CQG **35**, 035015 (2018)

$$\eta(\oplus, \text{☾}) = (-3 \pm 5) \times 10^{-14}$$

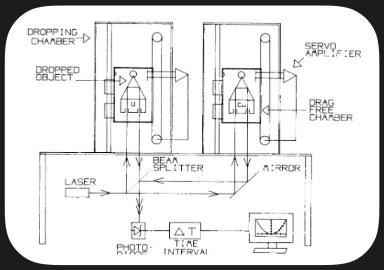


©CNES/ill./DUCROS David, 2016

MICROSCOPE

Touboul et al., PRL **119**, 231101 (2017)

$$\eta(\text{Ti,Pt:Rh}) = (-1 \pm 9(\text{stat}) \pm 9(\text{syst})) \times 10^{-15}$$



©Niebauer et al.

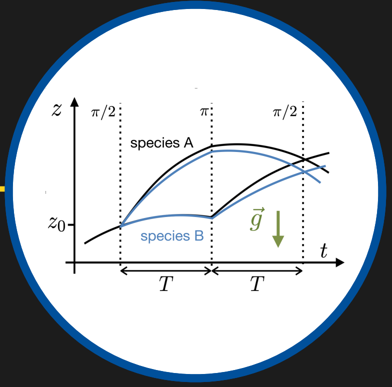
Laser gravimeter

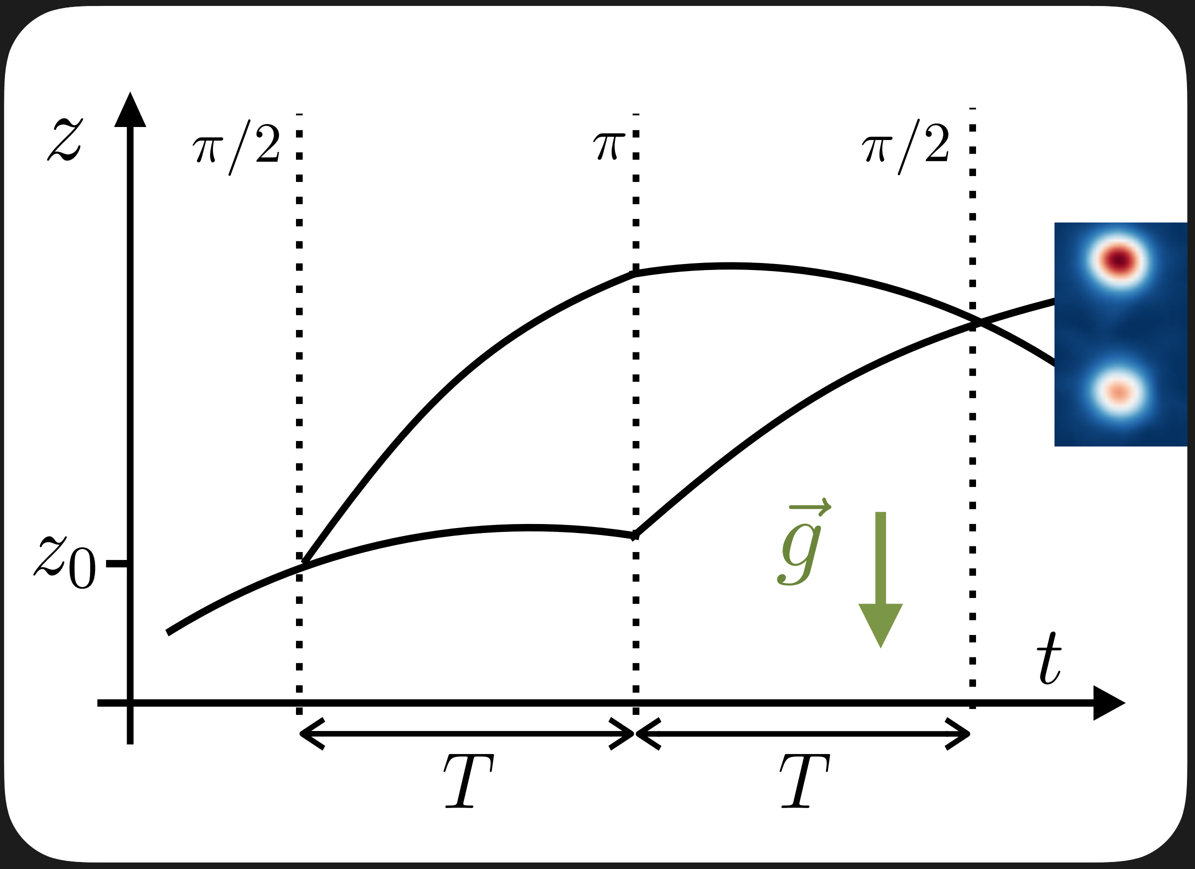
Niebauer et al., PRL **59**, 609 (1987)

$$\eta(\text{Cu,U}) = 5 \times 10^{-10}$$

©NASA

$$\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2}$$





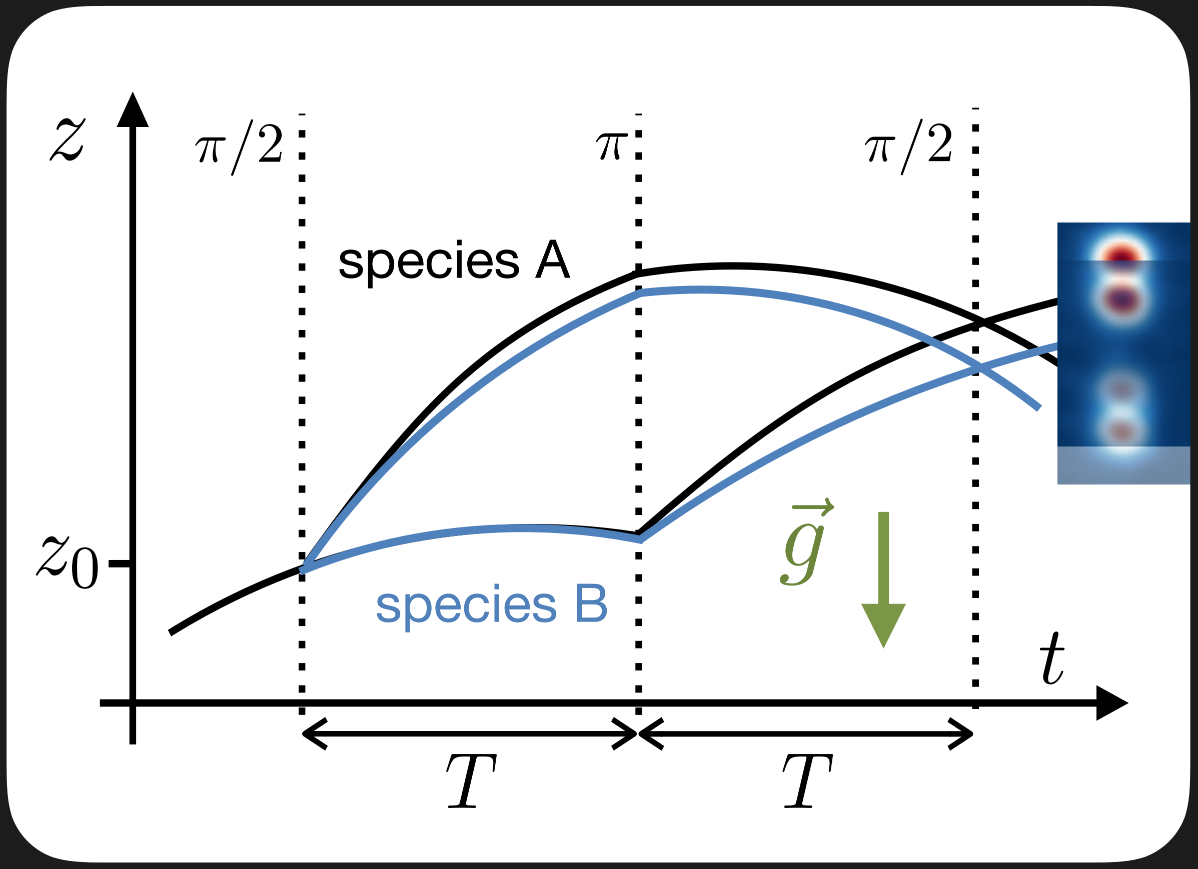
Population in output port

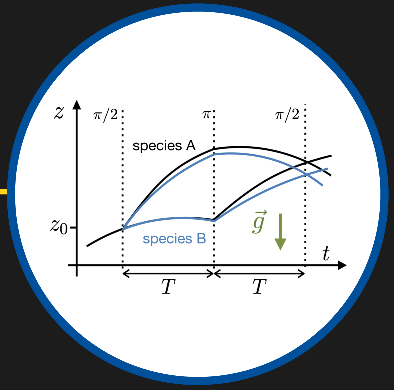
$$P \propto \cos(\Delta\phi)$$

with phase difference

$$\Delta\phi = \overrightarrow{k_{eff}} \cdot \overrightarrow{g} T^2$$

II - Atom Interferometry





Population in output port

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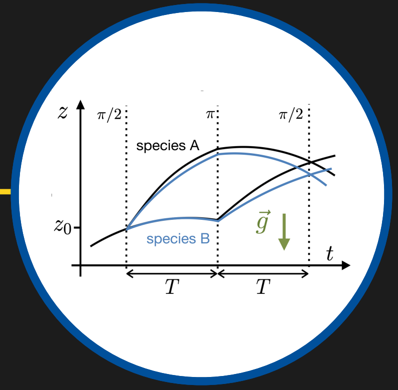
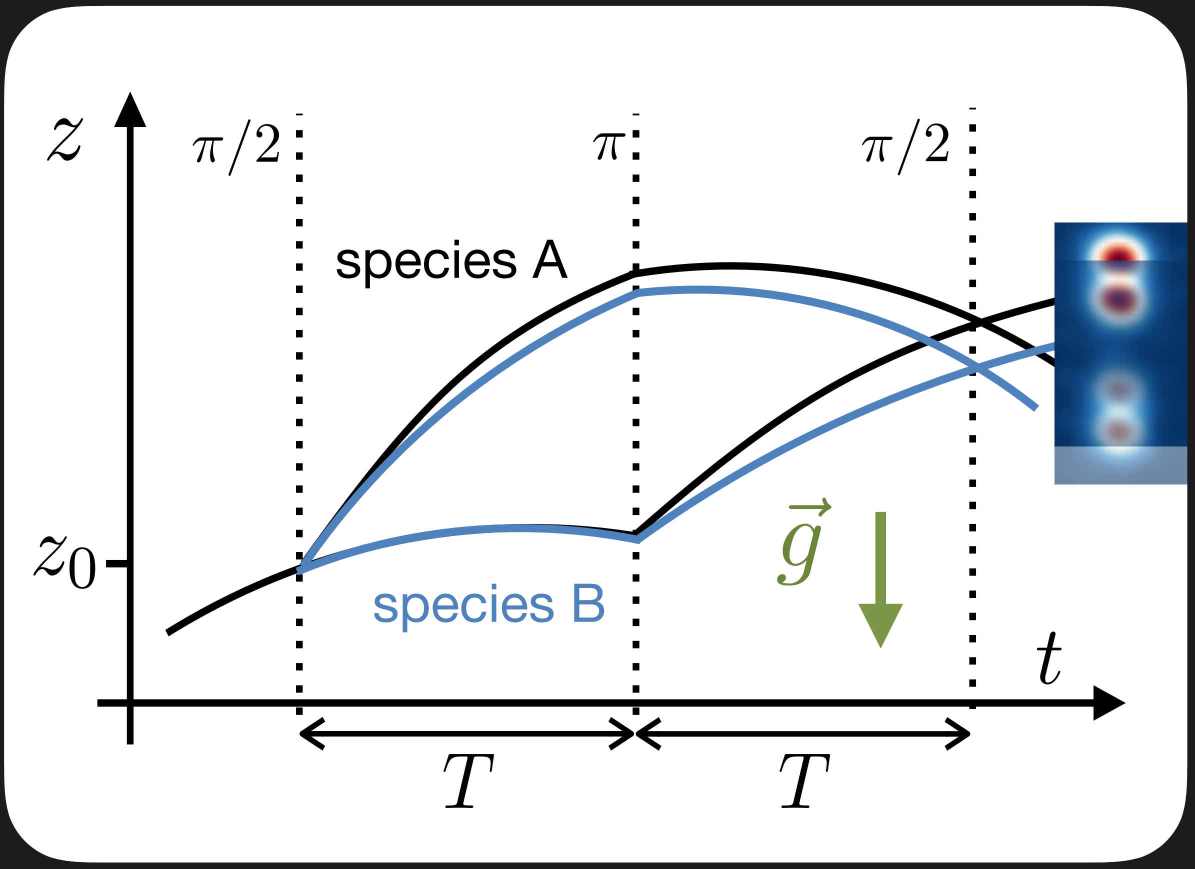




Population in output port

$$P \propto \cos(\Delta\phi)$$

with phase difference

$$\Delta\phi = \overrightarrow{k_{eff}} \cdot \vec{g}\, T^2$$

Simultaneous AIs measure differential acceleration:

acceleration of species i →

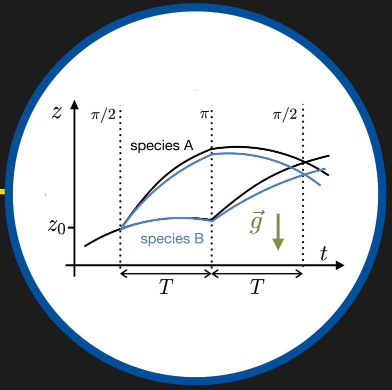
$$a_i = \frac{\Delta\phi_i}{k_{\mathrm{eff,i}} T_i^2}$$

← phase of species i

← scale factor of species i

$$\eta = 2\frac{|a_1 - a_2|}{a_1 + a_2}$$

II - Atom Interferometry



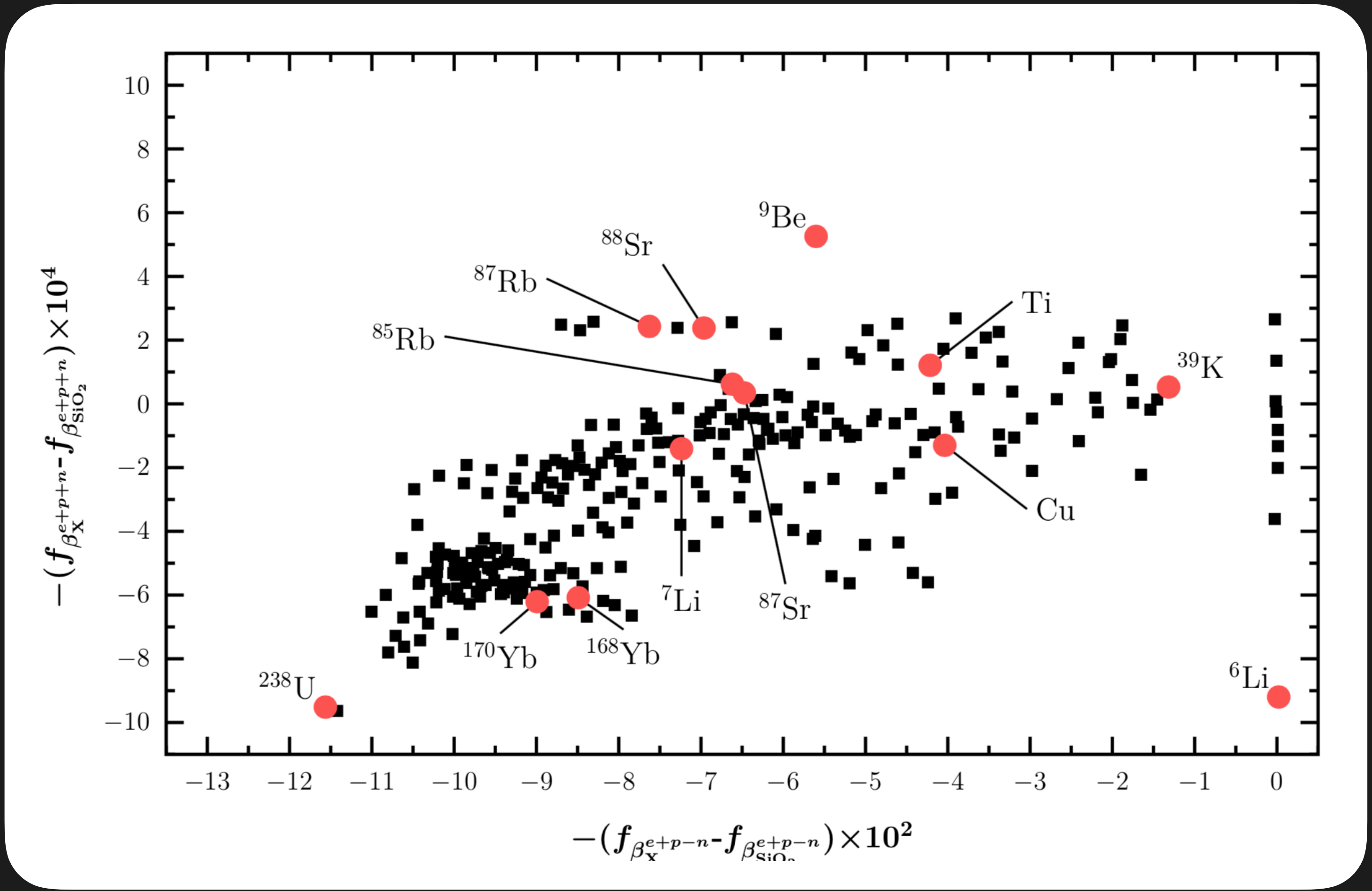
Quantum tests:

- range of test masses
- spin-gravity coupling
- large test mass coherence length

For example, test

- SME extension or
- dilaton model violation parameters:

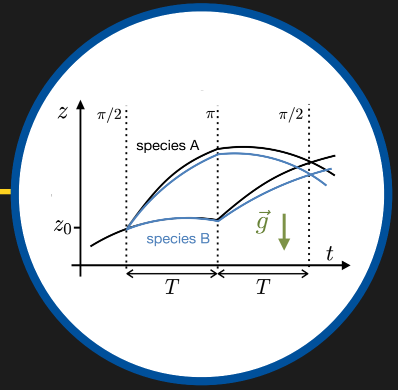
D. Schlippert, PhD Thesis, LUH (2014)



Schlippert et al., PRL 112, 203002 (2012)

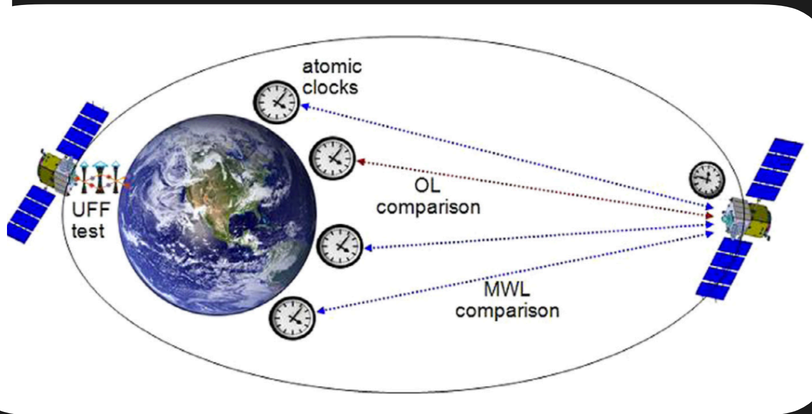
A	B	References	$Q'^1_A - Q'^1_B$ $\times 10^4$	$Q'^2_A - Q'^2_B$ $\times 10^4$	$f_{\beta_A^{e+p-n}} - f_{\beta_B^{e+p-n}}$ $\times 10^2$	$f_{\beta_A^{e+p+n}} - f_{\beta_B^{e+p+n}}$ $\times 10^4$	$f_{\beta_A^{\bar{e}+\bar{p}-\bar{n}}} - f_{\beta_B^{\bar{e}+\bar{p}-\bar{n}}}$ $\times 10^5$	$f_{\beta_A^{\bar{e}+\bar{p}+\bar{n}}} - f_{\beta_B^{\bar{e}+\bar{p}+\bar{n}}}$ $\times 10^4$
^{9}Be	Ti	[4]	−15.46	−71.20	1.48	−4.16	−0.24	−16.24
^{85}Rb	^{87}Rb	[11–13]	0.84	−0.79	−1.01	1.81	1.04	1.67
^{87}Sr	^{88}Sr	[14]	0.42	−0.39	−0.49	2.04	10.81	1.85
^{39}K	^{87}Rb	(this work)	−6.69	−23.69	−6.31	1.90	−62.30	0.64

II - Atom Interferometry



	Type	Ref.	A	B	δη(A,B)
Classical	MICROSCOPE	[0]	Ti	PtRh	~ 9×10^{-15}
	Lunar laser ranging	[1]	Earth	Moon	~1×10^{-14}
	Torsion balance	[2]	Be	Ti	~1×10^{-13}
	Laser gravimeter	[3]	Cu	U	~5×10^{-10}
Hybrid	AI+FG5 (Palo Alto)	[4]	^{133}Cs	FG5	~7×10^{-9}
	AI+FG5 (Paris)	[5]	^{87}Rb	FG5	~6×10^{-8}
Quantum	AI (Munich)	[6]	^{85}Rb	^{87}Rb	~2×10^{-7}
	AI (Palaiseau)	[7]	^{85}Rb	^{87}Rb	~3×10^{-7}
	BO (Firenze)	[8]	^{87}Sr	^{88}Sr	~2×10^{-7}
	AI (Wuhan)	[9]	^{85}Rb	^{87}Rb	~3×10^{-8}
	AI (IQ)	[10]	^{39}K	^{87}Rb	~5×10^{-7}

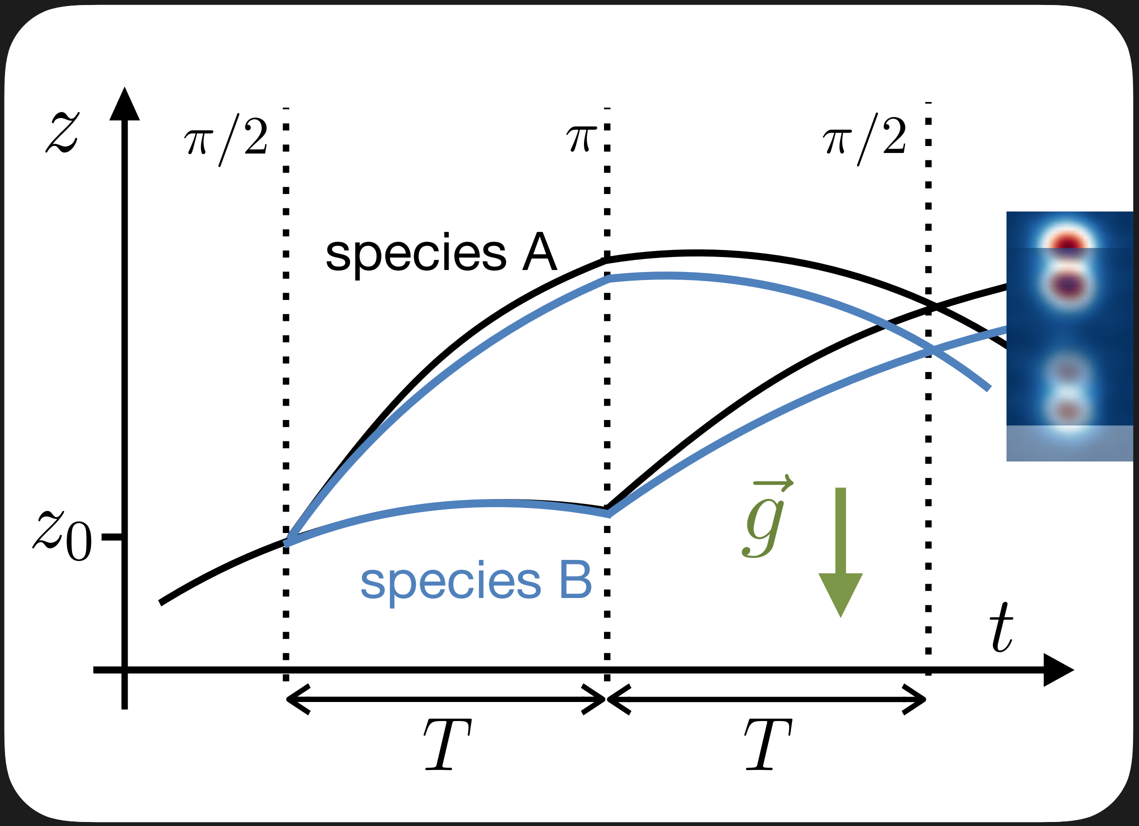
Tests of the WEP

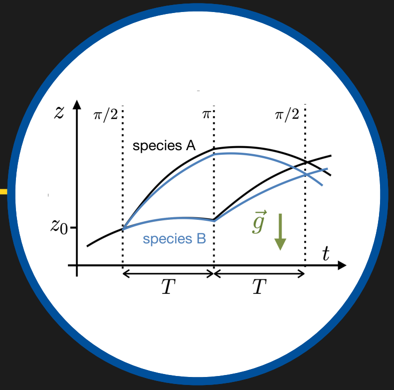


projected inaccuracy of large baseline [11,12] and space-borne [13] AI tests of the WEP: $\delta\eta \sim 10^{-13} - 10^{-15}$

[0] Touboul et al., PRL 119, 231101 (2017) **[1]** Hofmann et Müller, CQG 35, 035015 (2018); Williams et al., CQG 29, 184004 (2012); **[2]** Wagner et al., CQG 29, 184002 (2012); **[3]** Niebauer et al., PRL 59, 609 (1987); **[4]** Peters et al., Nature 400, 849 (1999); **[5]** Merlet et al., Metrologia 47, L9 (2010); **[6]** Fray et al., PRL. 93, 240404 (2004); **[7]** Bonnin et al., PRA 88, 043615 (2013); **[8]** Tarallo et al., PRL. 113, 023005 (2014); **[9]** Zhou et al., PRL 115, 013004 (2015); **[10]** Schlippert et al., PRL 112, 203002 (2014); **[11]** Hartwig et al., NJP 17, 035011 (2015),; **[12]** Overstreet et al., PRL 120, 183604 (2018), **[13]** Altschul et al., Adv. in Space Research 55(1), 2015 **see also:** ^{87}Rb F=1, F=2 & superposition (~10^{-9}): Rosi et al., NatComm 8, 15529 (2017); ^{87}Rb F=1 & F=2 ($3\cdot10^{-10}$): Zhang et al., arXiv: 1805.07758

III The Co-location problem

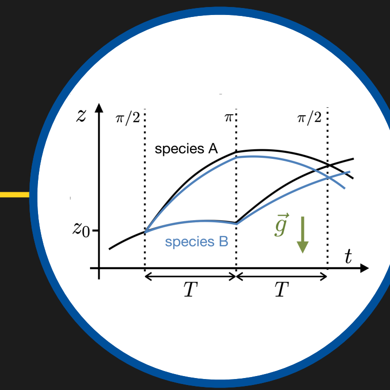




In presence of gravity gradients $\Gamma_{i,j} = \partial_j g_i$, initial coordinates $z_0, v_{z,0}$ enter the phase:

$$\Delta\phi = \vec{k}_{\mathrm{eff}} \cdot \vec{a}\, T^2 + k_{\mathrm{eff}} \Gamma_{zz} T^2 (z_0 + v_{z,0} T) + \ldots$$

III The Co-location problem



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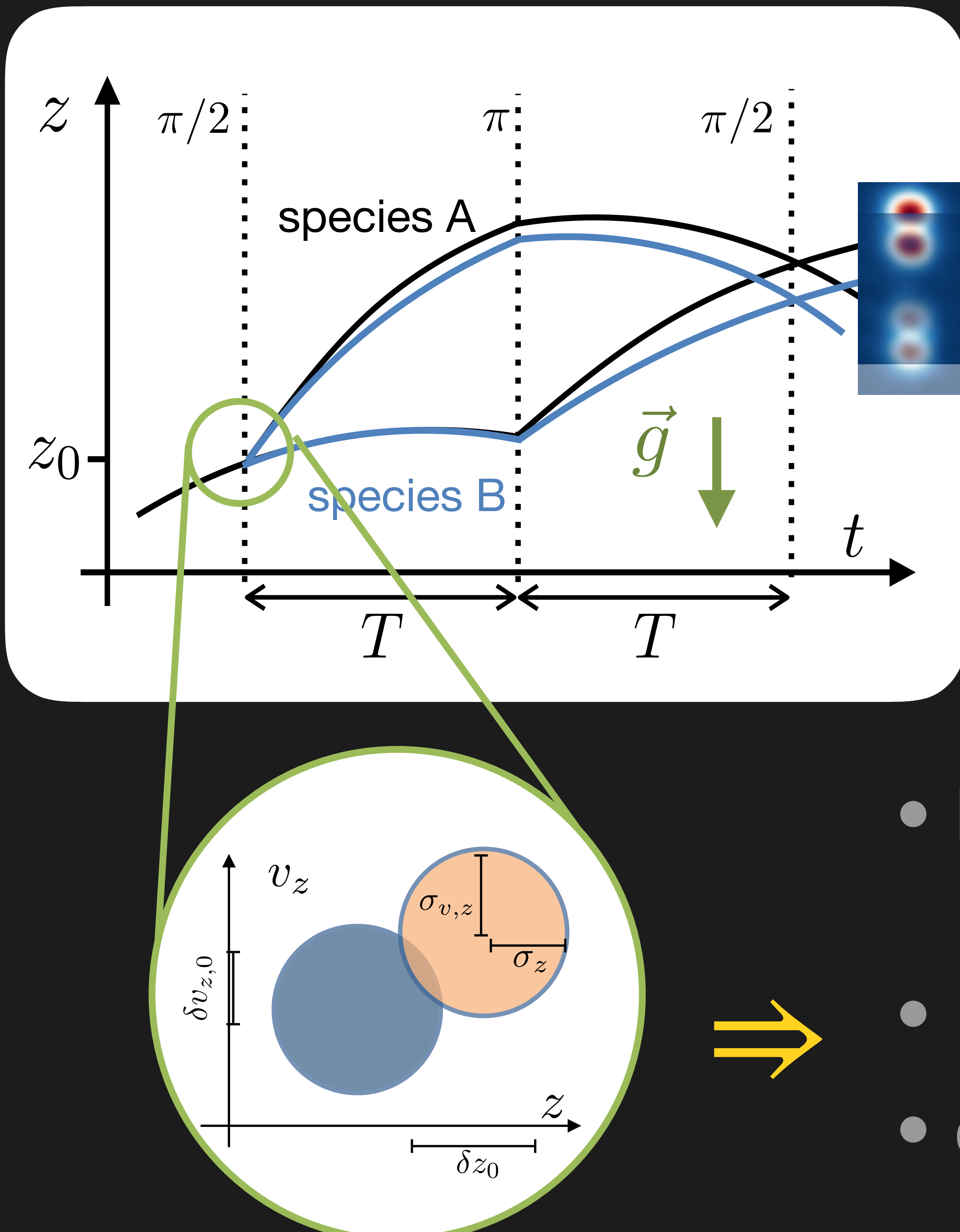


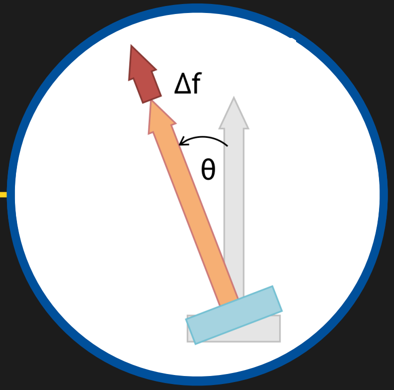
Phase-space uncertainties

$$\Delta\phi = \vec{k}_{\mathrm{eff}} \cdot \vec{a}\, T^2 + k_{\mathrm{eff}} \Gamma_{zz} T^2 (z_0 + v_{z,0} T) + \ldots$$

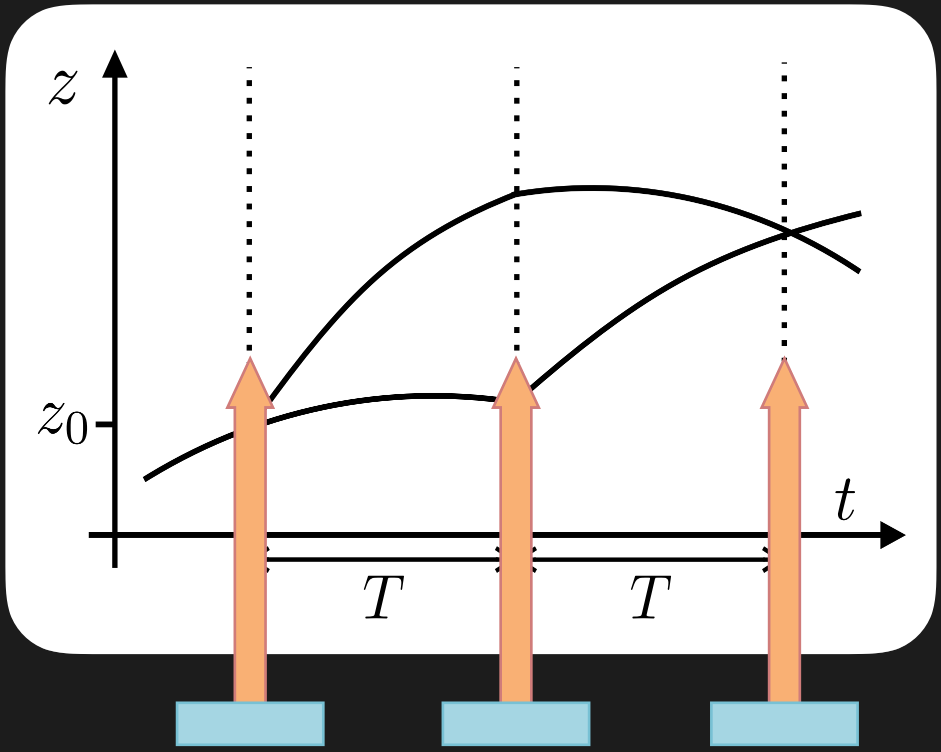
$\Rightarrow$

- Phase/acceleration measurement uncertainty
- loss of contrast
- one of major limitations in state of the art atomic gravimeters
- **δη ~10^{-15}: requires δr_0~nm, δv_0~nm/s**

IV Gravity gradient cancellation



Idea: Introduce controllable phase-shift to remove dependencies on initial coordinates

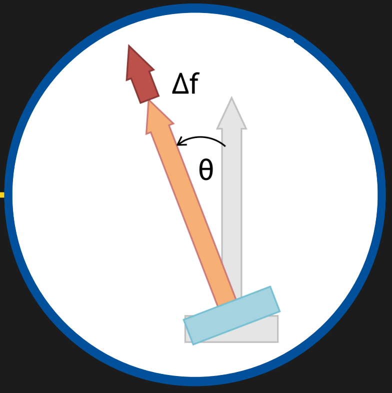


$$\mathbf{k}_{\text{eff}}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix} \quad \mathbf{k}_{\text{eff}}^{(2)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix} \quad \mathbf{k}_{\text{eff}}^{(3)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix}$$

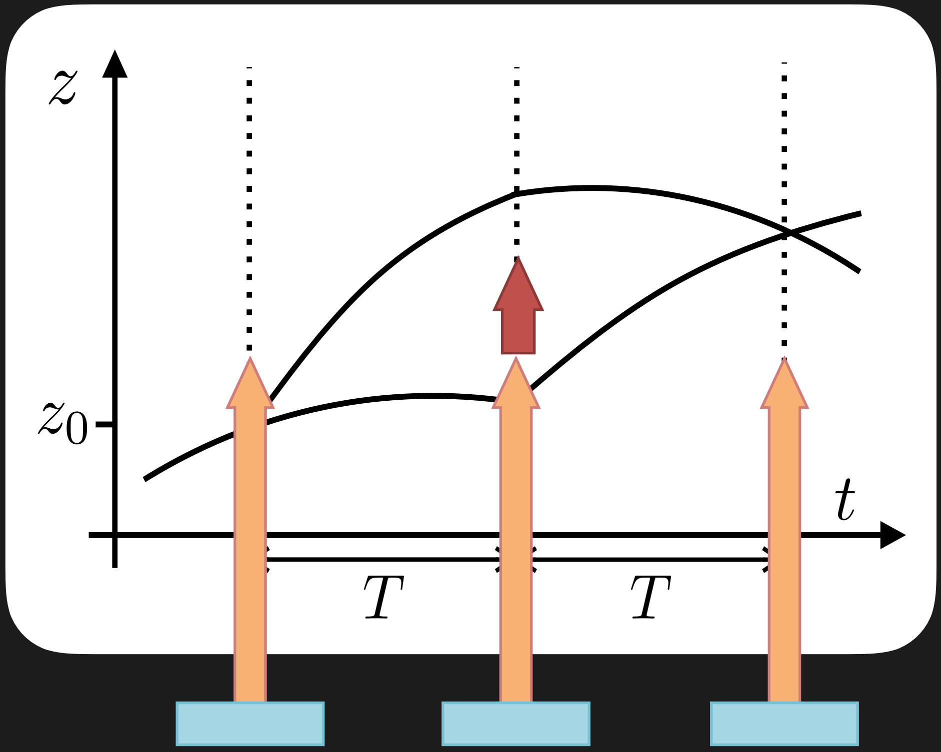
$$\Delta\phi = \ldots + k_{\text{eff}}\Gamma_{zz}T^2(z_0 + v_{z,0}T) \qquad + \ldots$$

Theory: Roura, PRL **118,** 160401 (2017); *Experiment*: Overstreet et al., PRL **120**, 183604 (2018); D'Amico et al., PRL **119**, 253201 (2017)

IV Gravity gradient cancellation



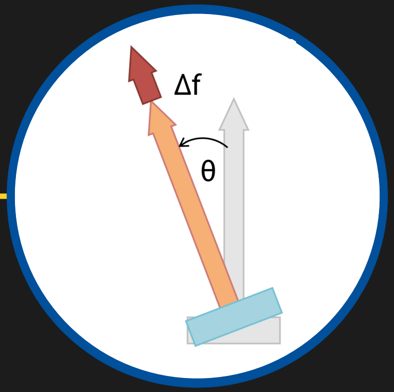
Idea: Introduce controllable phase-shift to remove dependencies on initial coordinates



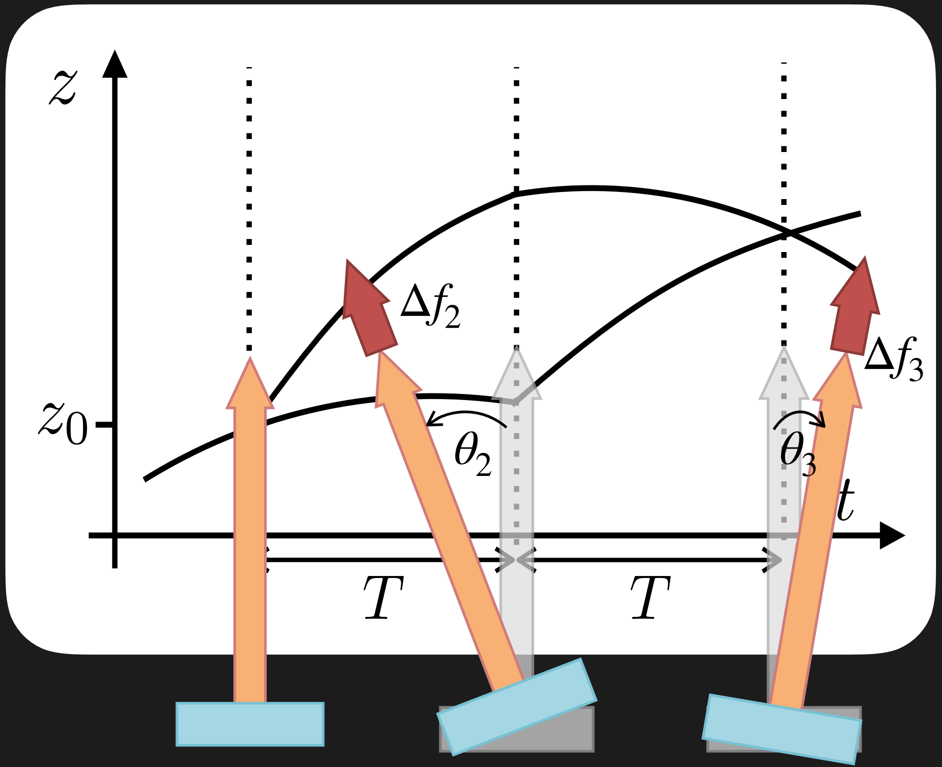
$$\mathbf{k}_{\text{eff}}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix} \quad \mathbf{k}_{\text{eff}}^{(2)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}}(1+\Delta k) \end{pmatrix} \quad \mathbf{k}_{\text{eff}}^{(3)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix}$$

$$\Delta\phi = \ldots + k_{\text{eff}}\Gamma_{zz}T^2(z_0 + v_{z,0}T) - 2\Delta k(z_0 + v_{z,0}T) + \ldots$$

Theory: Roura, PRL **118,** 160401 (2017); *Experiment*: Overstreet et al., PRL **120**, 183604 (2018); D'Amico et al., PRL **119**, 253201 (2017)



Idea: Introduce controllable phase-shift to remove dependencies on initial coordinates

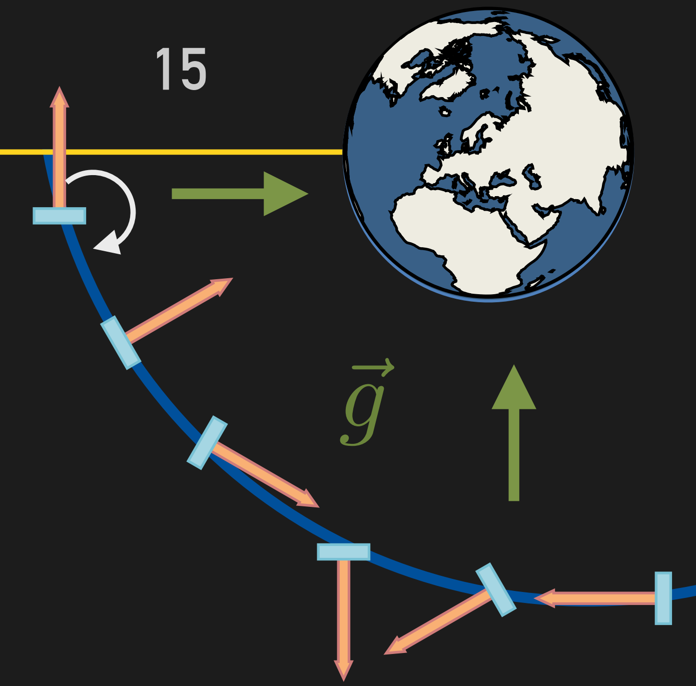


$$\mathbf{k}_{\text{eff}}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix} \quad \vec{k}_{\text{eff}}^{(2)} = k_{\text{eff}} \begin{pmatrix} \Delta_{2,x} \\ 0 \\ 1 + \Delta_{2,z} \end{pmatrix} \quad \vec{k}_{\text{eff}}^{(3)} = k_{\text{eff}} \begin{pmatrix} \Delta_{3,x} \\ 0 \\ 1 + \Delta_{3,z} \end{pmatrix}$$

$$\Delta\phi = \Delta\phi_0 + \sum_{j=x,y,z} \alpha_j r_j(0) + \beta_j \dot{r}_j(0)$$

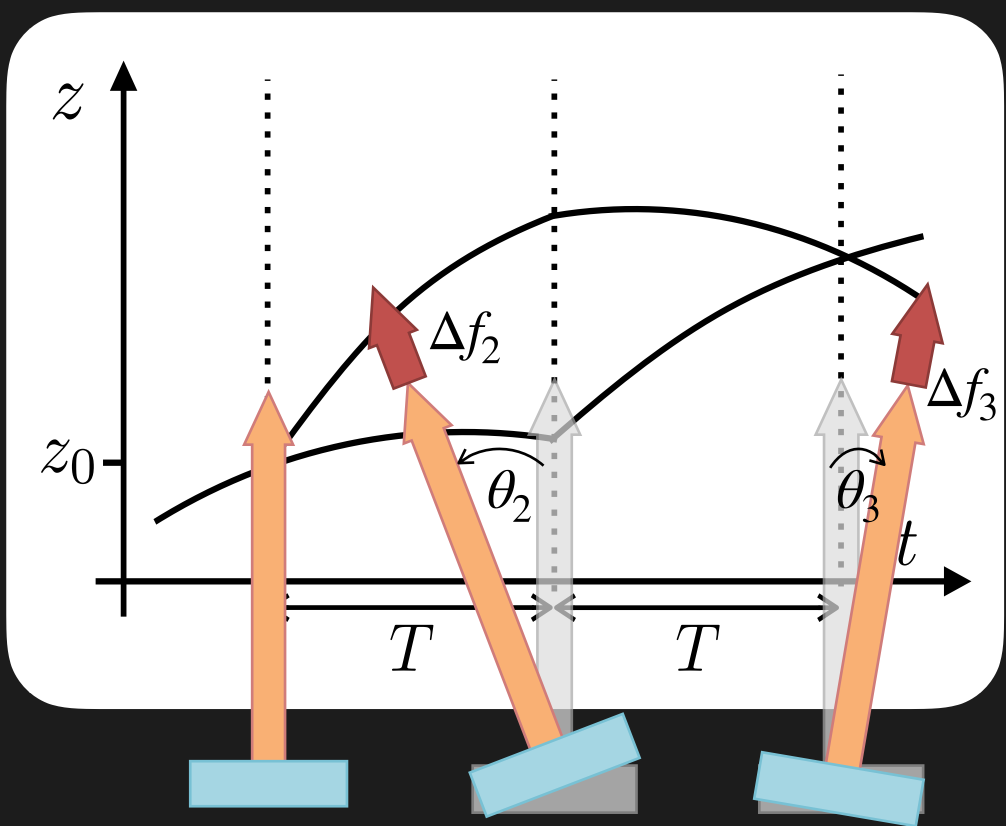
α_j, β_j : functions of $\Delta_{i,j}(\theta_i, \Delta f_i)$

IV Gravity gradient cancellation



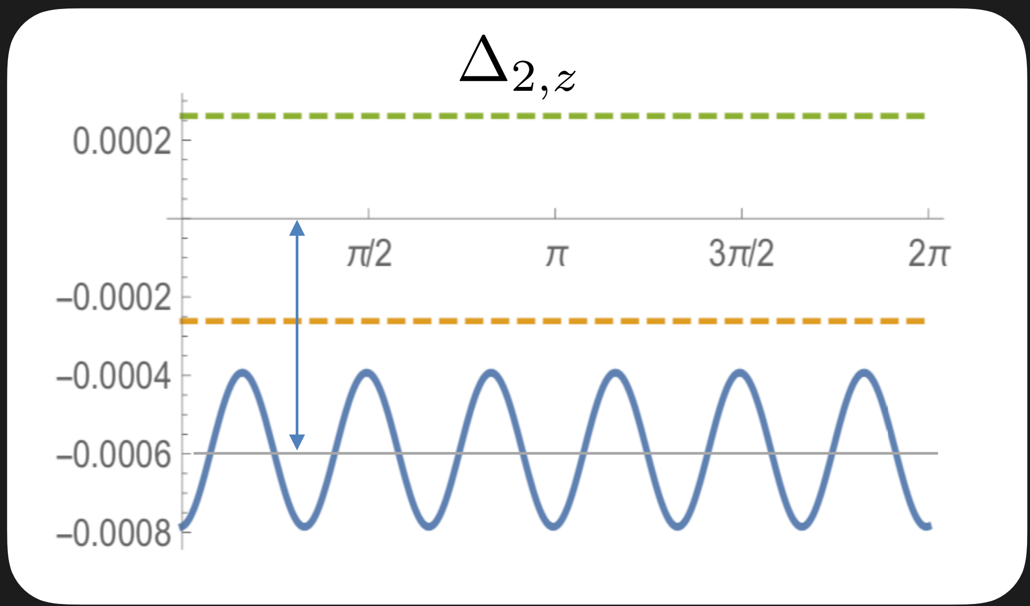
Application:

- Varying gravity gradient values
- Rotating systems



$$\mathbf{k}_{\text{eff}}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ k_{\text{eff}} \end{pmatrix} \quad \vec{k}_{\text{eff}}^{(2)} = k_{\text{eff}} \begin{pmatrix} \Delta_{2,x} \\ 0 \\ 1 + \Delta_{2,z} \end{pmatrix} \quad \vec{k}_{\text{eff}}^{(3)} = k_{\text{eff}} \begin{pmatrix} \Delta_{3,x} \\ 0 \\ 1 + \Delta_{3,z} \end{pmatrix}$$

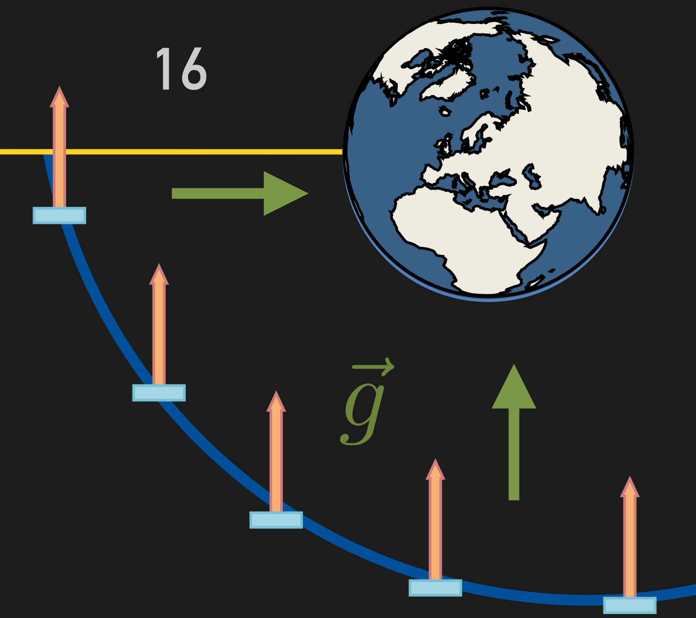
Example: Satellite spinning in orbital plane ($\Omega = 2 \text{x} \Omega_{\text{orbit}}$)



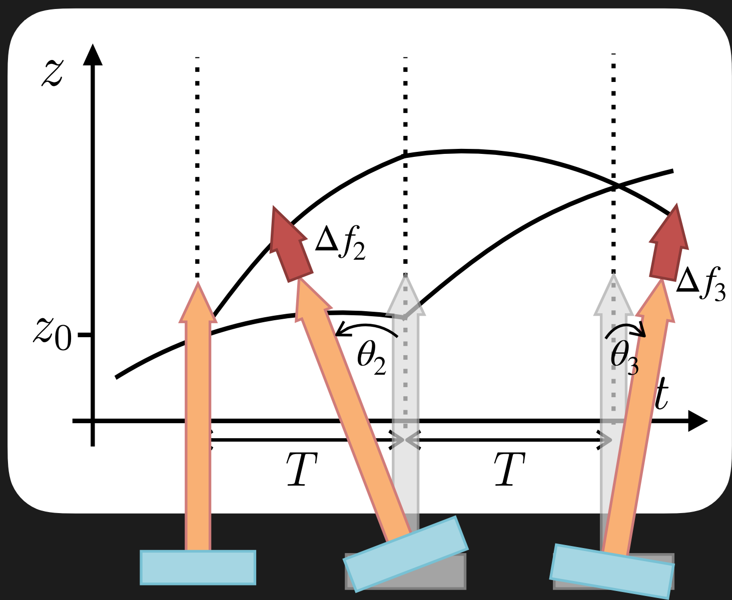
offset: rotation compensation by counter-rotation

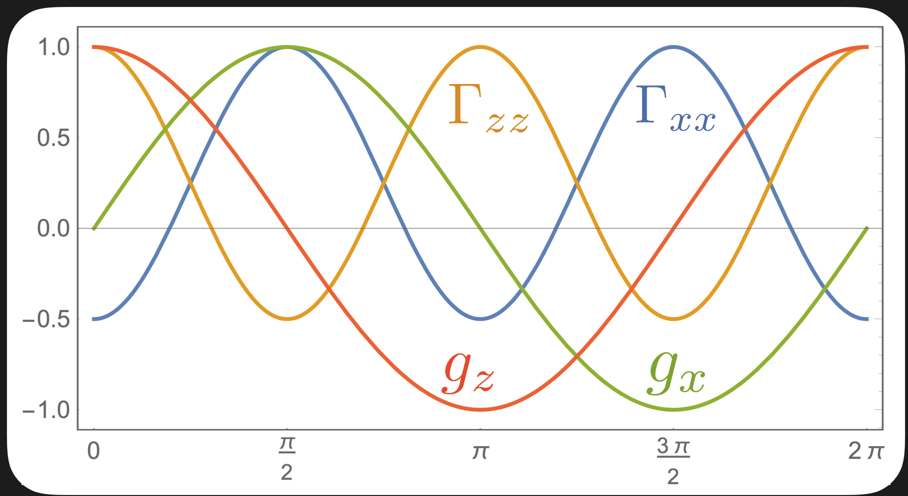
modulation: gravity gradient compensation

IV Gravity gradient cancellation



Feasibility analysis: Inertial mission

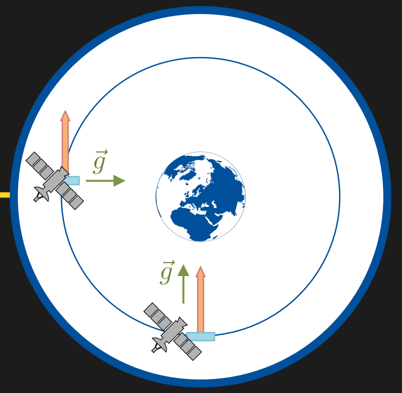




requirements for a 10^3 supression of gravity gradient related differential acceleration uncertainties

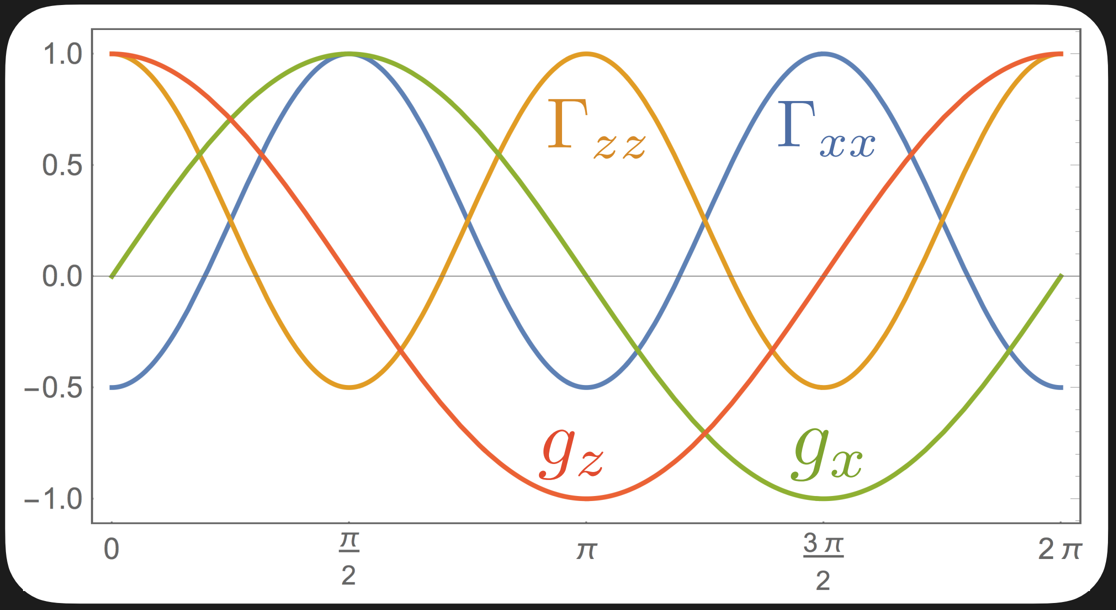
quantitiy	value	uncertainty	value	comment
θ	20μrad	$\delta\theta$	100nrad	max. laser tilt
Δf	10GHz	δf	10MHz	max. laser frequency shift
γ	$-2 \times 10^{-6}\text{s}^{-2}$	$\delta\gamma$	$-2 \times 10^{-9}\text{s}^{-2}$	gravity gradient
T	5s	δT	10μs	pulse separation time
Ω_y	0	$\delta\Omega$	10nrad	satellite spinning

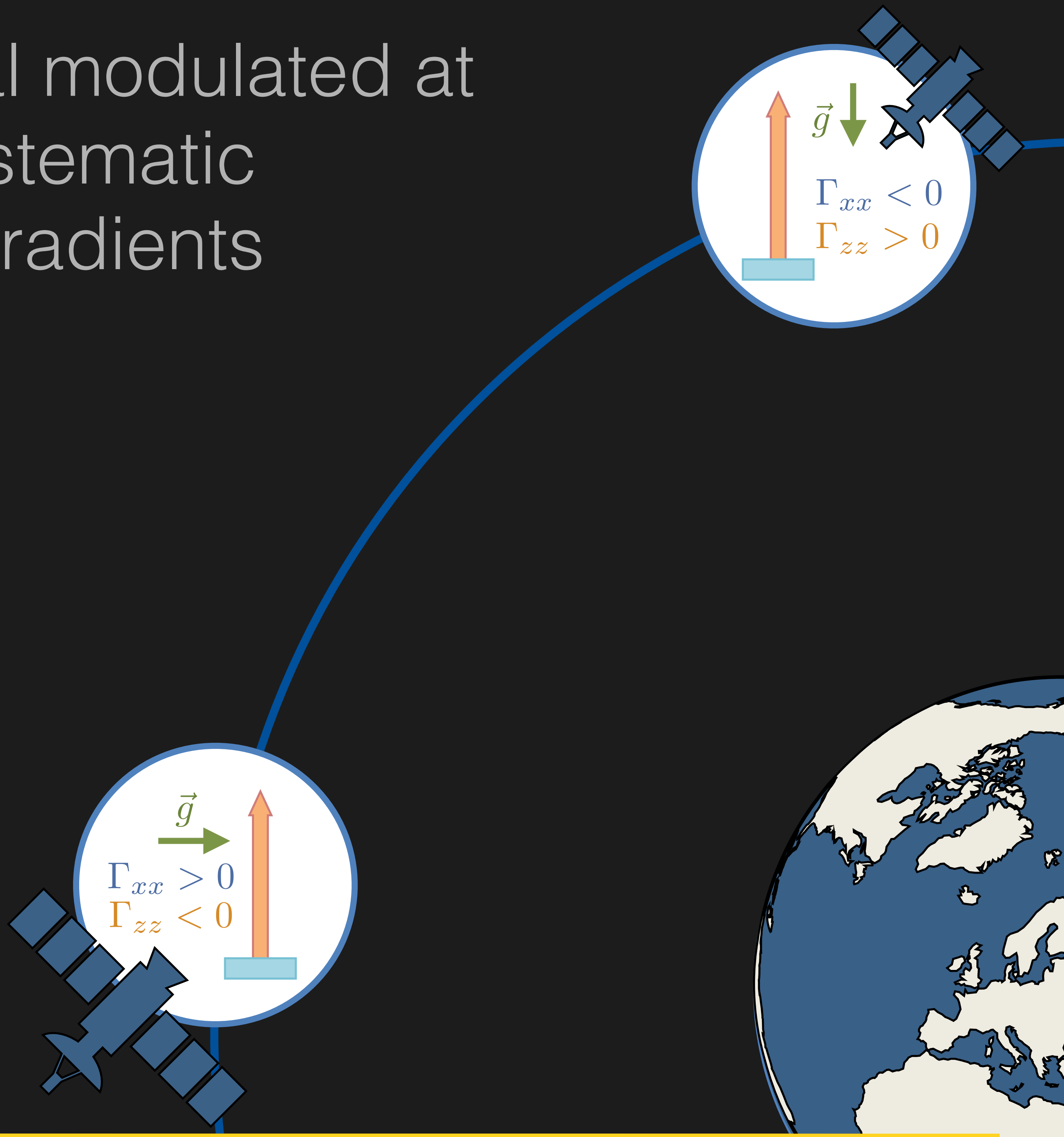
4 Demodulation



How to get rid of residual uncertainties?

Idea: Possible violation signal modulated at different frequency than systematic uncertainties from gravity gradients

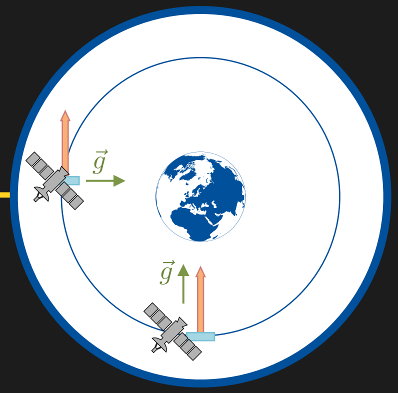




Wiliams et al., NJP **16**(12), (2017)

Touboul et al., PRL **119** (23), (2017)

4 Demodulation

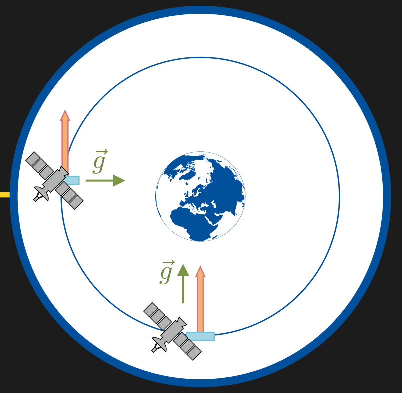


Differential acceleration signal:

$$\Delta a = \delta a \cos(\Omega_m t) + \Delta a_{\mathrm{const}} + \sum_{j=1} \Delta a^j_{\mathrm{sys}} \cos(j\Omega_m t),$$

poss. UFF-violation modulated at Ω_m

frequency components of syst. uncertainties



Differential acceleration signal:

$$\Delta a = \delta a \cos(\Omega_m t) + \Delta a_{\text{const}} + \sum_{j=1} \Delta a^j_{\text{sys}} \cos(j\Omega_m t),$$

poss. UFF-violation modulated at Ω_m

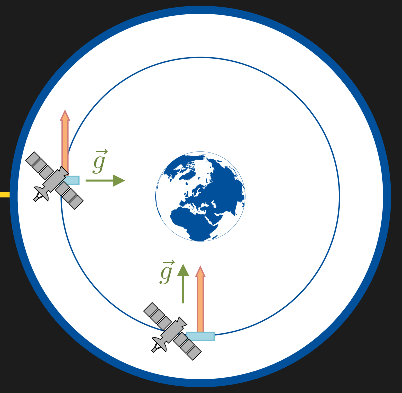
frequency components of syst. uncertainties

Demodulation at target frequency:

$$\frac{2}{\tau}\int_0^\tau \Delta a \cos(\Omega_m t)dt = (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\left[\frac{\delta a + \Delta a^1_{sys}}{2}\sin(2\Omega_m\tau) + \Delta a_{const}\sin(\Omega_m\tau)\right.$$

$$\left. + 2\sum_{j=2} \Delta a^j_{sys}\left[\frac{\sin([j\Omega_m - \Omega_m]\tau)}{j\Omega_m - \Omega_m} + \frac{\sin([j\Omega_m + \Omega_m]\tau)}{j\Omega_m - \Omega_m}\right]\right]$$

$$\leq (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\left[\frac{\delta a + \Delta a^1_{sys}}{4} + |\Delta a_{const}| + \frac{2}{3}\sum_{j=2} |\Delta a^j_{sys}|\right]$$



Differential acceleration signal:

$$\Delta a = \delta a \cos(\Omega_m t) + \Delta a_{\text{const}} + \sum_{j=1} \Delta a^j_{\text{sys}} \cos(j\Omega_m t),$$

poss. UFF-violation modulated at Ω_m

frequency components of syst. uncertainties

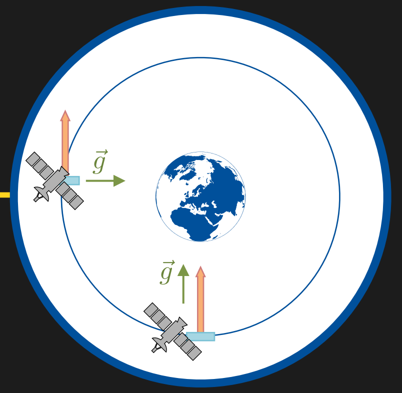
Demodulation at target frequency:

$$\frac{2}{\tau}\int_0^\tau \Delta a \cos(\Omega_m t) dt = (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\Bigg[\frac{\delta a + \Delta a^1_{sys}}{2}\sin(2\Omega_m\tau) + \Delta a_{const}\sin(\Omega_m\tau)$$

$$+ 2\sum_{j=2}\Delta a^j_{sys}\left[\frac{\sin([j\Omega_m - \Omega_m]\tau)}{j\Omega_m - \Omega_m} + \frac{\sin([j\Omega_m + \Omega_m]\tau)}{j\Omega_m - \Omega_m}\right]\Bigg]$$

$$\leq (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\left[\frac{\delta a + \Delta a^1_{sys}}{4} + |\Delta a_{const}| + \frac{2}{3}\sum_{j=2}|\Delta a^j_{sys}|\right]$$

4 Demodulation



Differential acceleration signal:

$$\Delta a = \delta a \cos(\Omega_m t) + \Delta a_{\mathrm{const}} + \sum_{j=1} \Delta a^j_{\mathrm{sys}} \cos(j\Omega_m t),$$

poss. UFF-violation modulated at Ω_m

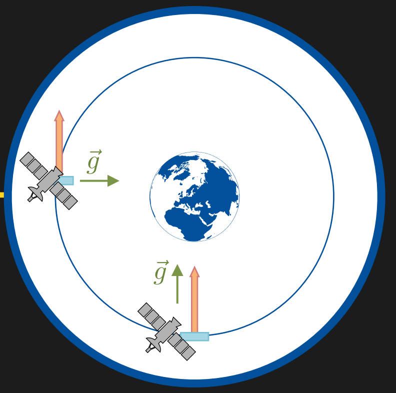
frequency components of syst. uncertainties

Demodulation at target frequency:

$$\frac{2}{\tau}\int_0^\tau \Delta a \cos(\Omega_m t)dt = (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\left[\frac{\delta a + \Delta a^1_{sys}}{2}\sin(2\Omega_m\tau) + \Delta a_{const}\sin(\Omega_m\tau)\right.$$

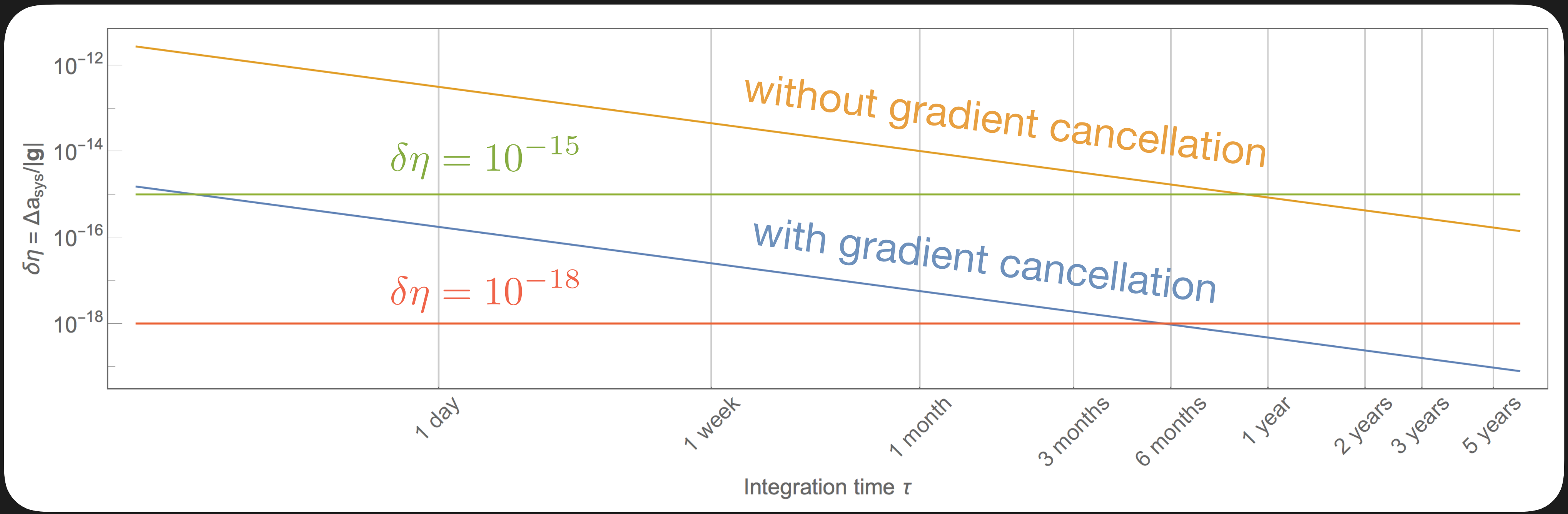
$$\left. + 2\sum_{j=2}\Delta a^j_{sys}\left[\frac{\sin([j\Omega_m - \Omega_m]\tau)}{j\Omega_m - \Omega_m} + \frac{\sin([j\Omega_m + \Omega_m]\tau)}{j\Omega_m - \Omega_m}\right]\right]$$

$$\leq (\delta a + \Delta a^1_{sys}) + \frac{2}{\tau\Omega_m}\left[\frac{\delta a + \Delta a^1_{sys}}{4} + |\Delta a_{const}| + \frac{2}{3}\sum_{j=2}|\Delta a^j_{sys}|\right]$$



Study case: WEP test with atom interferometry

- inertial configuration on circular orbit
- GGC: reduces verification time & improves contrast
- Demodulation: reduces residual uncertainties



assumptions:
δr_0 = 1µm
δv_0 = 1µm/s
altitude = 700km
$\delta\theta$ = 100nrad
δf = 10MHz
$\delta\Omega$ = 10nrad
$\delta\gamma = 10^{-3}\,\gamma$

10^{-18} supression of gravity gradient induced uncertainties with moderate requirements and integration time

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Quantum Tests

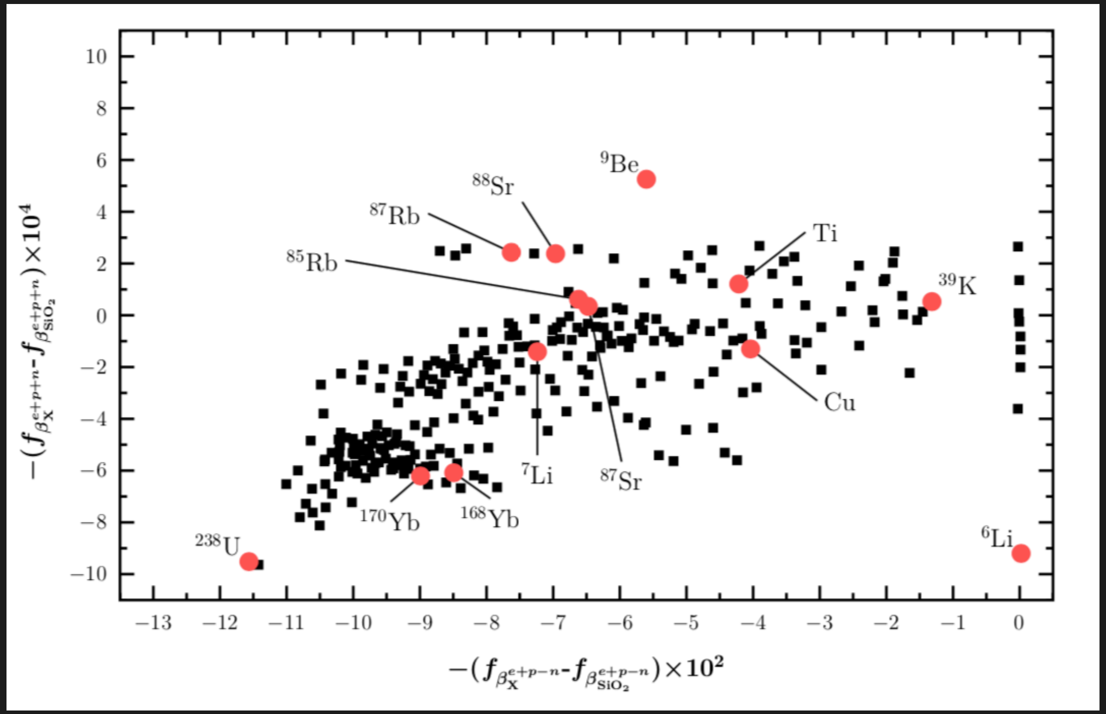
Dilaton model

$$\eta_{A,B} \cong D_1(Q'^1_A - Q'^1_B) + D_2(Q'^2_A - Q'^2_B)$$

Standard Model extension

$$\eta_{A,B} \cong \beta_A - \beta_B$$

$$\beta_X \equiv f_{\beta_X^{e+p-n}} \beta^{e+p-n} + f_{\beta_X^{e+p+n}} \beta^{e+p+n} + f_{\beta_X^{\bar{e}+\bar{p}-\bar{n}}} \beta^{\bar{e}+\bar{p}-\bar{n}} + f_{\beta_X^{\bar{e}+\bar{p}+\bar{n}}} \beta^{\bar{e}+\bar{p}+\bar{n}}$$



A	B	References	$Q'^1_A - Q'^1_B$ $\times 10^4$	$Q'^2_A - Q'^2_B$ $\times 10^4$	$f_{\beta_A^{e+p-n}} - f_{\beta_B^{e+p-n}}$ $\times 10^2$	$f_{\beta_A^{e+p+n}} - f_{\beta_B^{e+p+n}}$ $\times 10^4$	$f_{\beta_A^{\bar{e}+\bar{p}-\bar{n}}} - f_{\beta_B^{\bar{e}+\bar{p}-\bar{n}}}$ $\times 10^5$	$f_{\beta_A^{\bar{e}+\bar{p}+\bar{n}}} - f_{\beta_B^{\bar{e}+\bar{p}+\bar{n}}}$ $\times 10^4$
^{9}Be	Ti	[4]	−15.46	−71.20	1.48	−4.16	−0.24	−16.24
^{85}Rb	^{87}Rb	[11–13]	0.84	−0.79	−1.01	1.81	1.04	1.67
^{87}Sr	^{88}Sr	[14]	0.42	−0.39	−0.49	2.04	10.81	1.85
^{39}K	^{87}Rb	(this work)	−6.69	−23.69	−6.31	1.90	−62.30	0.64