ACES-PHARAO test of the gravitational redshift : refined estimation of the expected uncertainty

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Monday, October 22nd

Outline

Outline

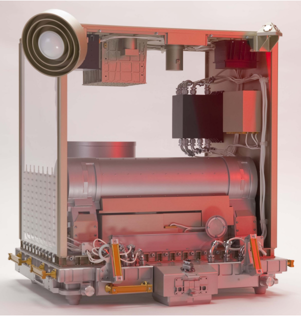
ACES-PHARAO mission

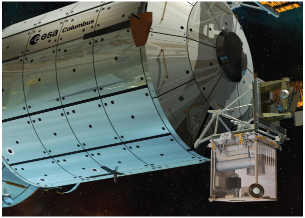
Objectives

Demonstrate ACES high performance and the ability to achieve high stability on space-ground time and frequency transfer.

Perform tests of fundamental physics at unprecedented accuracy

Ph Laurent et al. 2015. The ACES/PHARAO space mission





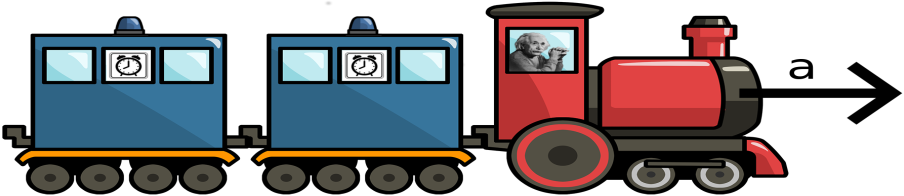
Launch date
Early 2020

Partners
CNES, ESA, industries and laboratories

Duration
18 months up to 3 years

For more details, see tomorrow's early conference by Luigi Cacciapuoti

Clocks desynchronisation



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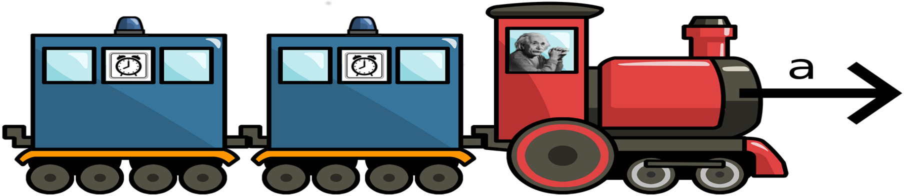

Doppler effect

$$\nu_i = \nu_0 \frac{1 + \frac{v_i}{c}}{1 + \frac{v_O}{c}} \tag{1}$$

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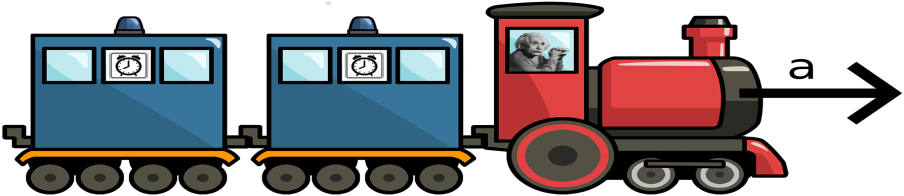

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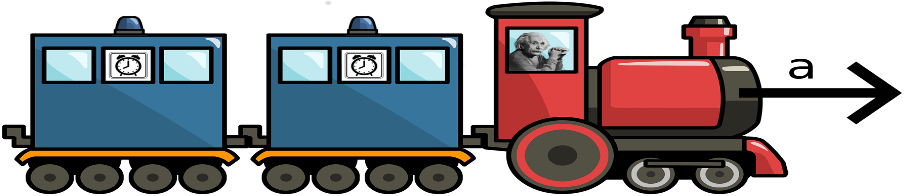
Relative frequency difference

$$\frac{\Delta\nu}{\nu} = \frac{\Delta v}{c} = \frac{a\Delta t}{c} = \frac{aL}{c^2} \tag{2}$$

Clocks desynchronisation



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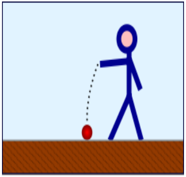
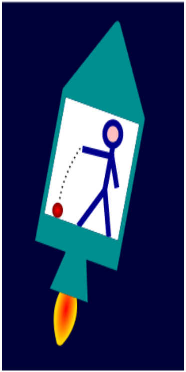
Desynchronisation

$$\Delta\tau = \int_0^t \frac{\Delta\nu}{\nu} \mathrm{d}t' = \int_0^t \frac{aL}{c^2} \mathrm{d}t' \simeq \frac{aL}{c^2} t \qquad (3)$$

Equivalence principle





Two people drop an object, the first one is in a gravitational field ($\vec{g}$), the other is in a rocket with a constant acceleration ($\vec{a} = -\vec{g}$).

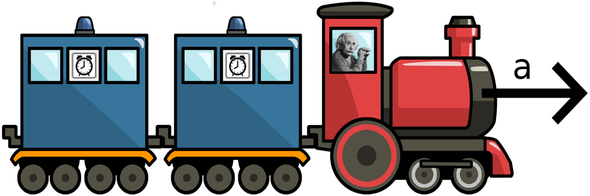
Equivalence principle

- we [...] assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system. -
Einstein, 1907

Equivalence principle decomposition :

- Universality of Free Fall [Joel Bergé presentation]
- Lorentz invariance [Helene Pihan-Le Bars presentation]
- Local position invariance

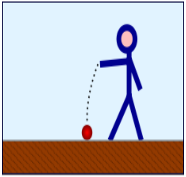
Clocks gravitational redshift

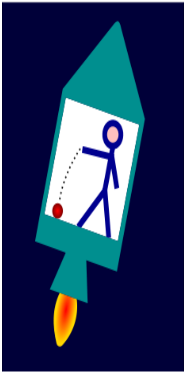


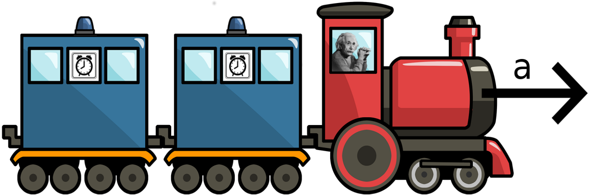
Desynchronisation : acceleration

$$\Delta\tau = \frac{aL}{c^2} t$$

Clocks gravitational redshift







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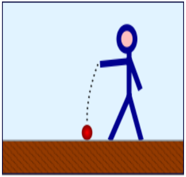

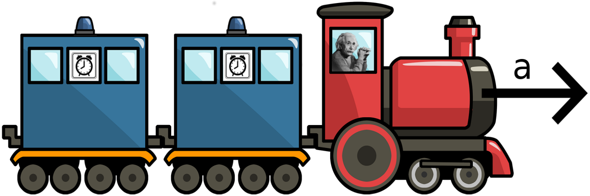

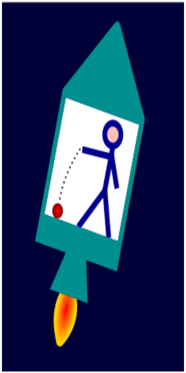

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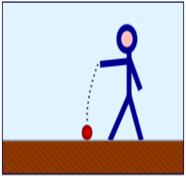
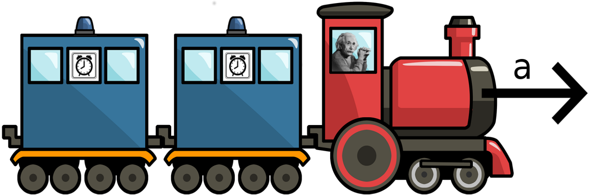
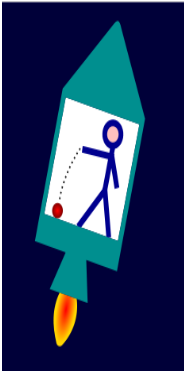






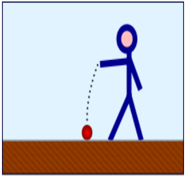
Desynchronisation : acceleration

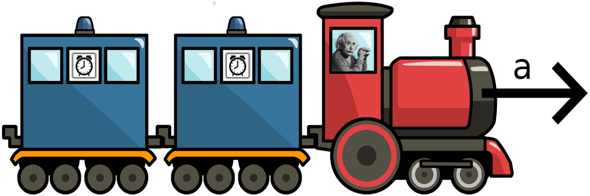
$$\Delta\tau = \frac{aL}{c^2} t$$

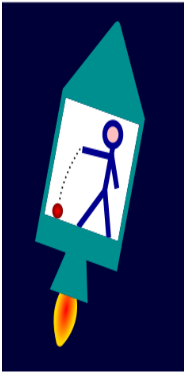
Clocks gravitational redshift

$$\Delta\tau \simeq \frac{gh}{c^2} t \qquad (4)$$

Clocks gravitational redshift







Desynchronisation : acceleration

$$\Delta\tau = \frac{aL}{c^2} t$$

Clocks gravitational redshift

$$\Delta\tau \simeq \frac{gh}{c^2} t \qquad (4)$$

$$\left.\frac{gh}{c^2}\right|_{BB} \Delta T \simeq 100ps \qquad \left.\frac{\Delta\nu}{\nu}\right._{ACES} \Delta T \simeq 10ps$$

Outline

Test of the gravitational redshift : experimental data

Desynchronisation model

$$\Delta\tau(t) = \Delta\tau_0 + \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2}\mathrm{d}t' + \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \quad (5)$$

Test of the gravitational redshift : experimental data

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Gravity Probe A

$$\sigma_\alpha = 2 \times 10^{-4}$$

R. F. C. Vessot et al. (1979)

Test of the gravitational redshift : experimental data

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GREAT Experiment

No spoiler !

See Pacôme's presentation

Test of the gravitational redshift : experimental data

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GREAT Experiment

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ACES-PHARAO

$$\sigma_\alpha = \mathrm{LOW} \times 10^{-6}$$

Test of the gravitational redshift : model

Fitted model

$$Y(t) = \Delta\tau(t) - \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2} \mathrm{d}t' - \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2} \mathrm{d}t' \quad (6)$$

Test of the gravitational redshift : model

Fitted model

$$
\begin{aligned}
Y(t) &= \Delta\tau(t) - \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2}\mathrm{d}t' - \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \\
&= \Delta\tau_0 + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t'
\end{aligned}
\tag{6}
$$

Test of the gravitational redshift : model

Fitted model

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\begin{aligned}
Y(t) &= \Delta\tau(t) - \int_{t_0}^{t} \frac{V_{ground}^2 - V_{space}^2}{2c^2}\mathrm{d}t' - \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \\
&= \Delta\tau_0 + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2}\mathrm{d}t' \\
&= \begin{pmatrix} 1 & \vdots & \vdots & \int_{t_0}^{t_1} \frac{\Delta U}{c^2}\mathrm{d}t' \\ \vdots & 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \int_{t_0}^{t_n} \frac{\Delta U}{c^2}\mathrm{d}t' \end{pmatrix} \begin{pmatrix} \Delta\tau_0^{OPMT} \\ \Delta\tau_0^{PTBB} \\ \vdots \\ \boxed{\alpha} \end{pmatrix}
\end{aligned}
\tag{6}
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Test of the gravitational redshift : model

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&= AX
\end{aligned} \tag{6}
$$

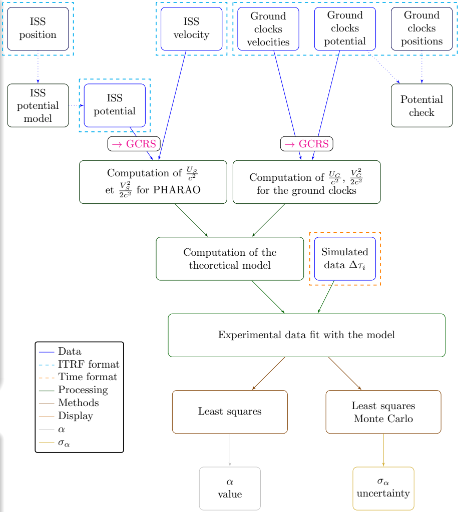
Test of the gravitational redshift : software

Analysis software

- Needs the ISS/PHARAO orbitography file, the ground clocks position and EOP
- Remove the known effect to the experimental/simulated data
- Create the model matrix
- Perform least squares methods to evaluate α and its uncertainty σ_α

Simulation software

- Create data gaps
- Add noise



Outline

Analysis methods

Method	Least squares Monte Carlo (LSMC)	Generalized least squares (GLS)
Model	$Y = Ax$	$WY = WAx$ with W the inverse noise covariance matrix
Solution	$x = (A^T A)^{-1} A^T Y$	$x = (A^T WA)^{-1} A^T WY$
Uncertainty	$\sigma_{LS} = \sigma_{noise}(A^T A)^{-1/2}$	$\sigma_{GLS} = (A^T WA)^{-1/2}$
How to get σ_α ?	Standard deviation of a set of N_{MC} least squares simulation	σ_{GLS}
CPU (time)	N_{MC} linear dependance	Instantaneous
RAM (memory)	Data length : n	Storing and inverting W : n^2
Prerequisites	General noise characteristics	Good knowledge of W
Validity area		inversion of a $n \times n$ covariance matrix $\Rightarrow n < 10.000$
Uncertainty	Theoretically higher than GLS	Cramér-Rao bound : best estimator
Summary	No-Brainer is Simpler : Computer efficient implementation but leads to higher uncertainty	Brainer is Better : Best estimator but requires to know the covariance matrix

Results

$$\sigma_{LSMC} \simeq \sigma_{GLS} \quad (7)$$

Phase or frequency model

Phase model

$$Y(t) = \Delta\tau_0 + \boxed{\alpha} \int_{t_0}^{t} \frac{U_{ground} - U_{space}}{c^2} \mathrm{d}t' \quad (8)$$

Frequency model

$$X(t) = \frac{dY}{dt} = \boxed{\alpha} \frac{U_{ground} - U_{space}}{c^2} \quad (9)$$

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	Phase	Frequency
σ_α	2.9×10^{-6}	3.0×10^{-4}

Table: Uncertainty of α : phase vs frequency

Phase or frequency model

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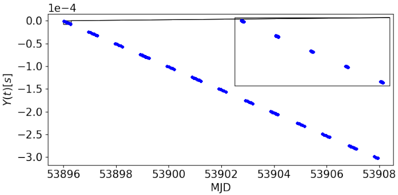
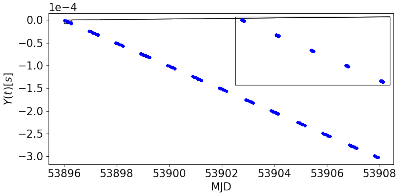
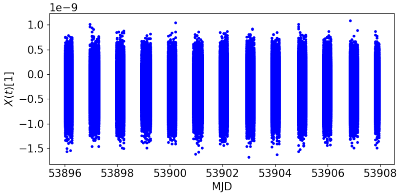


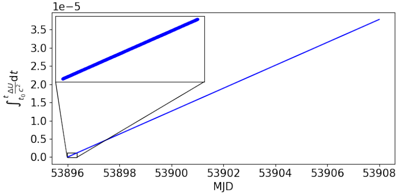
Data gaps

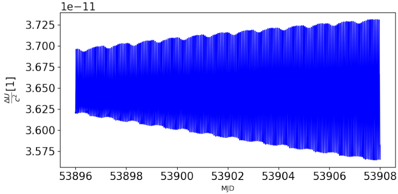
- 5 to 7 passes over a ground station per day
- 90 minutes orbital period
- 5 to 10 minutes pass duration

Phase or frequency model









α estimation

α is the slope of Y

$$\sigma_\alpha \propto \frac{\sigma_{RandomWalk}}{\Delta T} = \frac{1}{\sqrt{\Delta T}}$$

α estimation

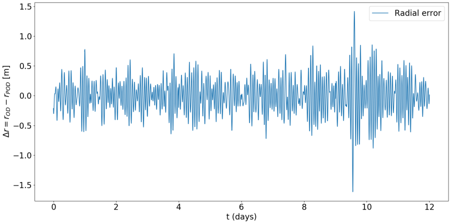
α is the mean of X

$$\sigma_\alpha \propto \sigma_{White} = \frac{1}{f\sqrt{\Delta T}}$$

ISS orbit deterioration

Scaling the ISS orbit error

- Precise orbit *POD*
- Less precise *OD*
- Orbit error $OD - POD$



Orbit file

$$OF = POD + k(OD - POD)$$

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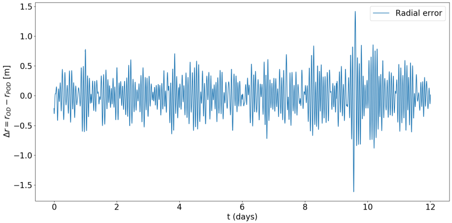


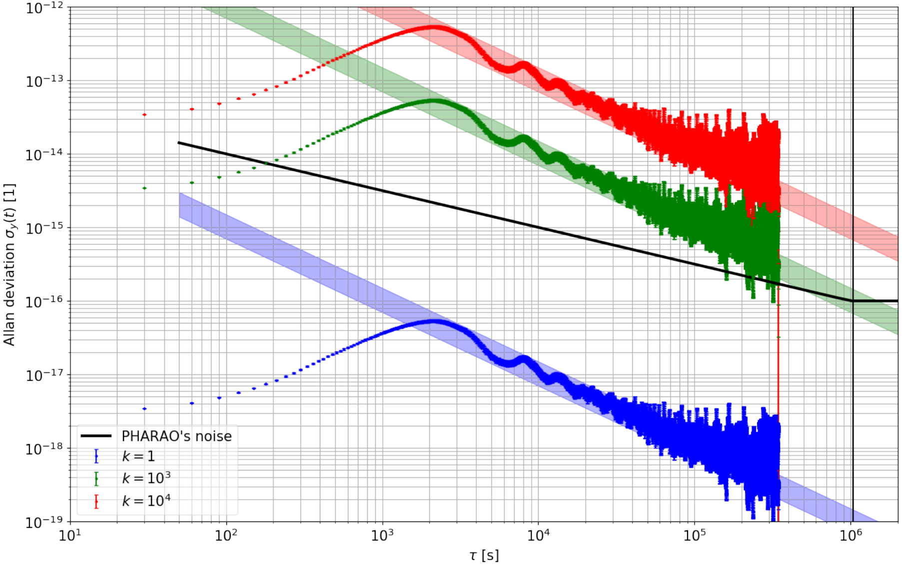
Orbit file

$$OF = POD + k(OD - POD)$$

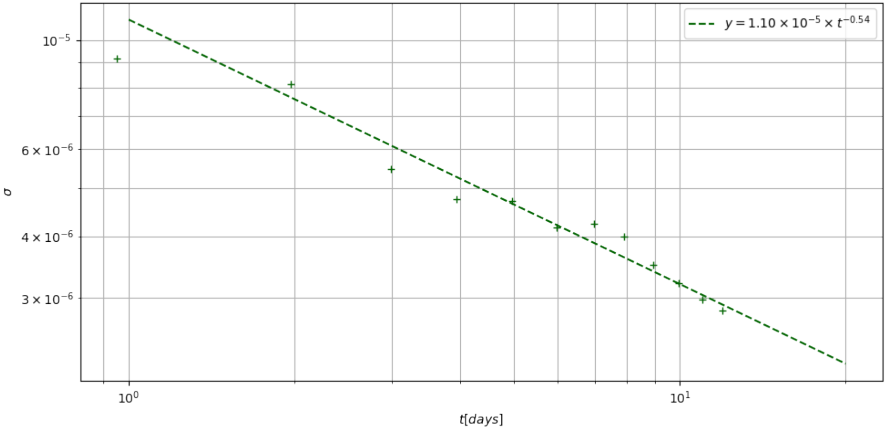
$k =$	0	1	10^3	10^4
Orbit error		1m	1km	10km
α	2×10^{-6}	2×10^{-6}	-1×10^{-6}	-3×10^{-5}
σ_α	4×10^{-6}	4×10^{-6}	4×10^{-6}	4×10^{-6}
Significant	False	False	False	True

Table: α : ISS orbit deterioration

ISS orbit deterioration



Expectation at 20 days



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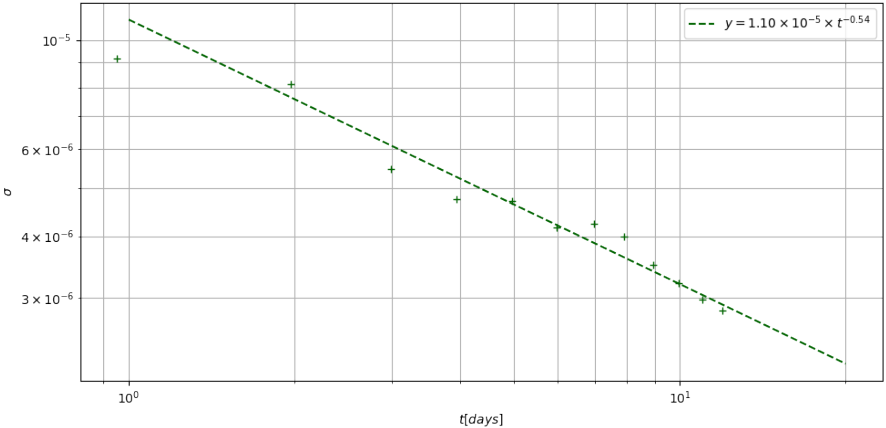

t	σ_α
20 days	2.2×10^{-6}

Table: σ_α at 20 days

Outline

Conclusion

Analysis methods

We will use the most efficient method (LSMC) rather than the best possible estimator (GLS).

Phase or frequency analysis

Due to the mission specificity, we will use the phase model over the frequency.

ISS orbit deterioration

A 100m error on the ISS orbit will not affect the gravitational redshift test.

Final expected uncertainty

We will achieve :

$$\sigma_\alpha = (2 - 3) \times 10^{-6} \tag{10}$$

Thank you for your attention