NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

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From Level-1B Instrument Data to Level-2 Spherical Harmonics

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New Refined Observations of Climate Change from Spaceborne Gravity Missions

From Level-1B Instrument Data to Level-2 Spherical Harmonics

… a 90 min ride through gravity field processing

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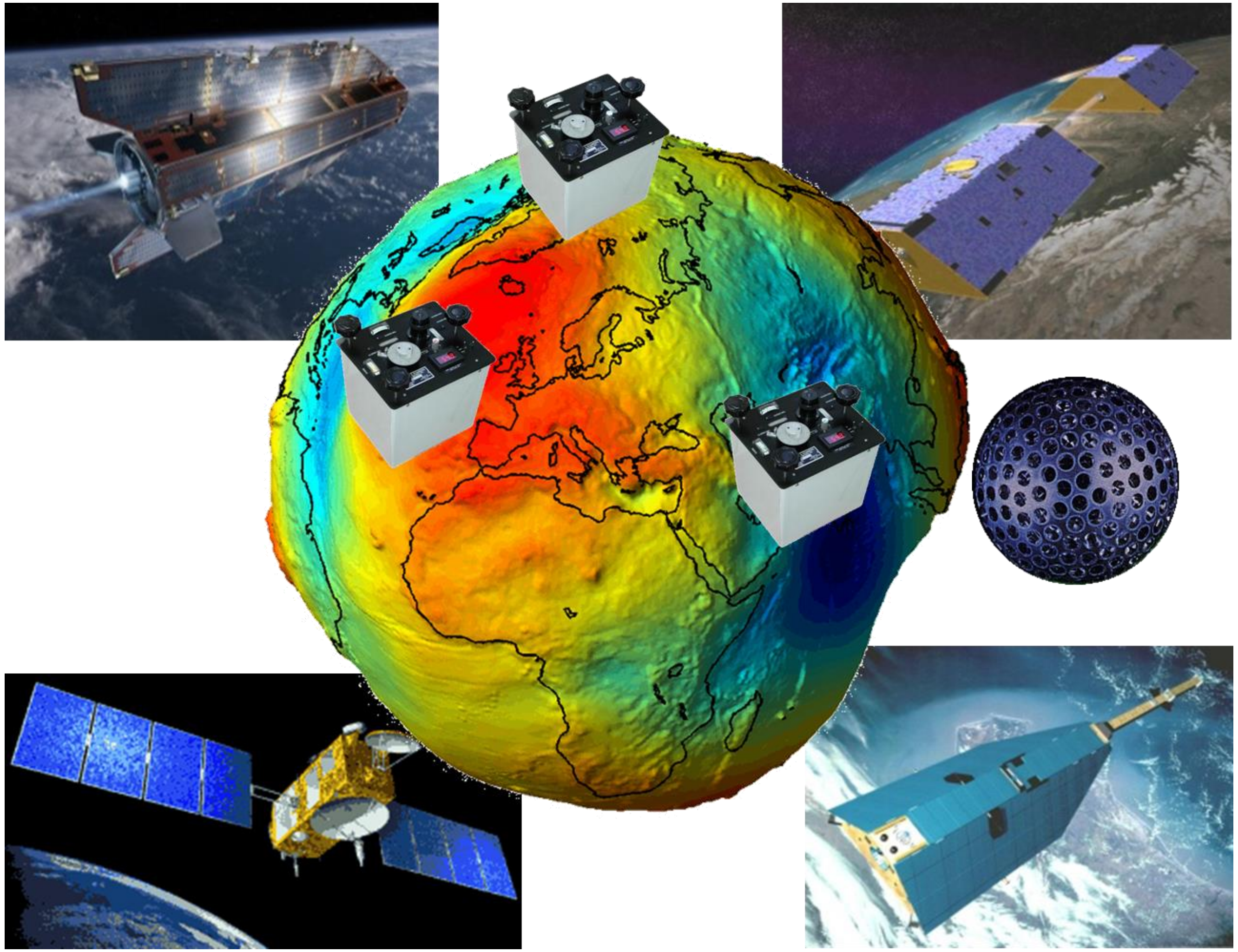
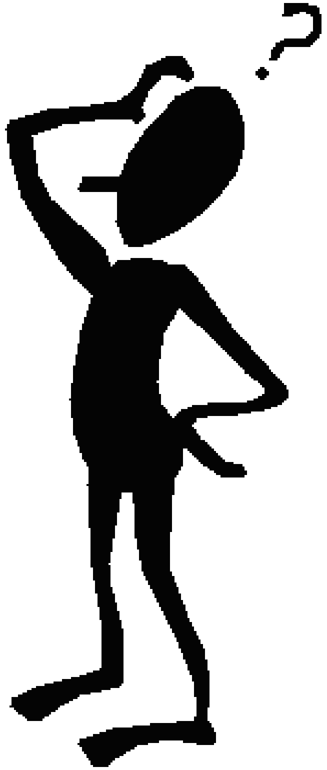
Contents

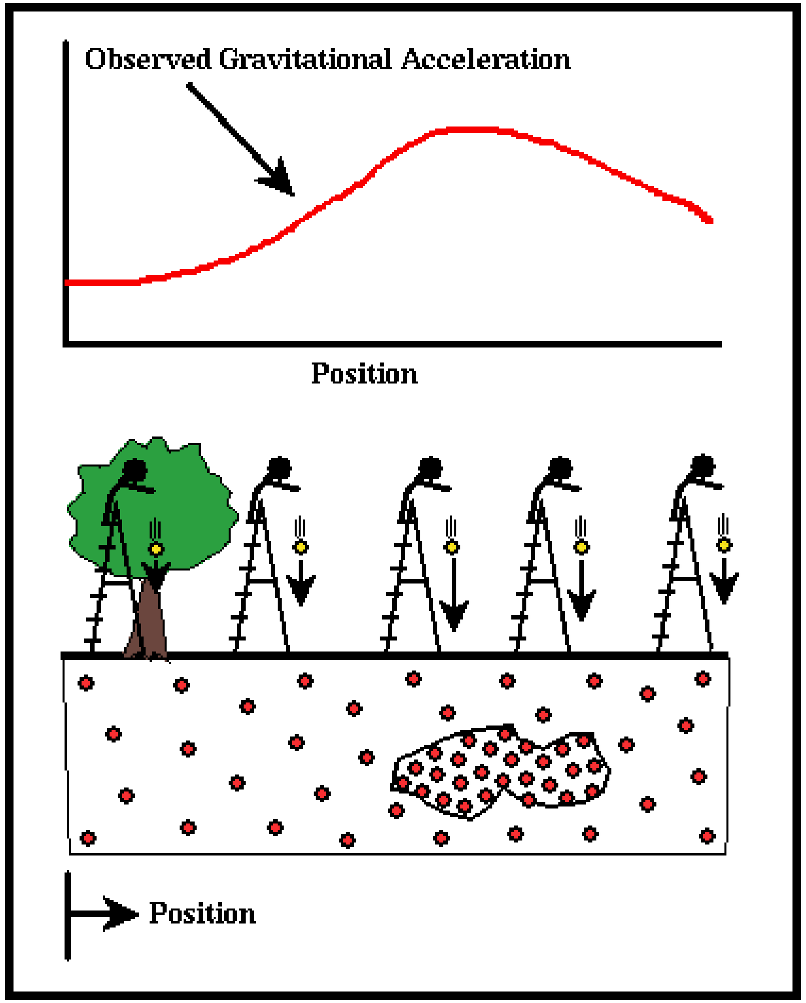
How can we observe the Earth's gravity field?





1. Basics: Earth Gravity Field (static, time-variable)
2. Observation Techniques

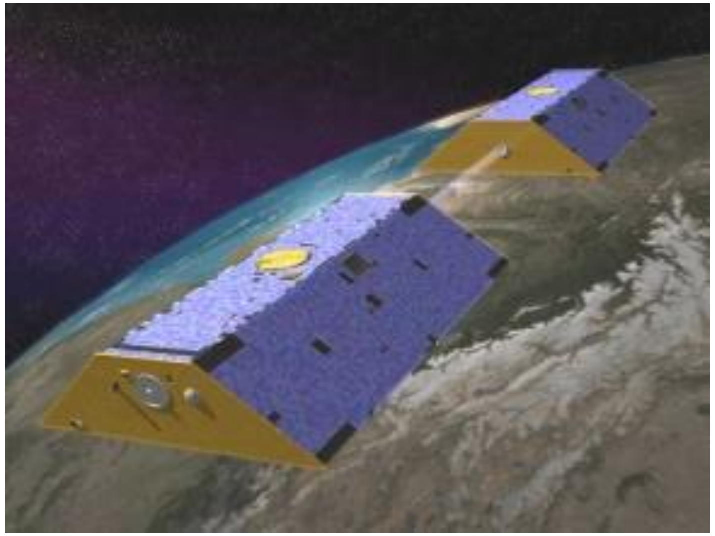
1. Mass and Gravity



1. Static vs. Time-variable Gravity Field

GOCE

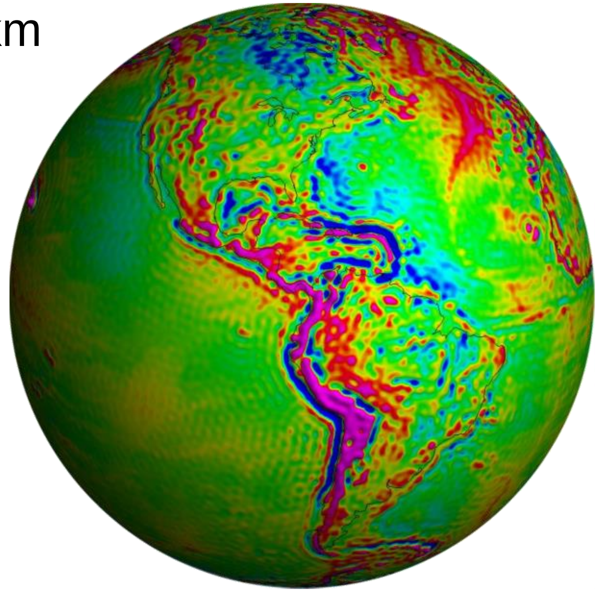
GRACE/ GRACE-FO

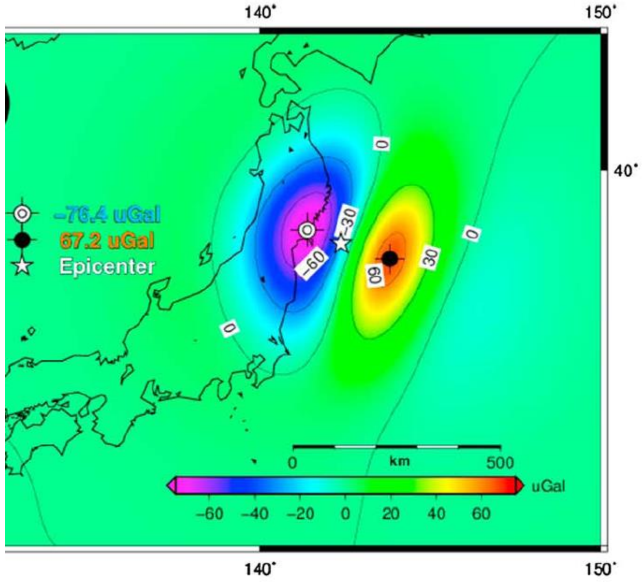


Static gravity field

– Spatial resolution >70 km

– Globally homogeneous accuracy



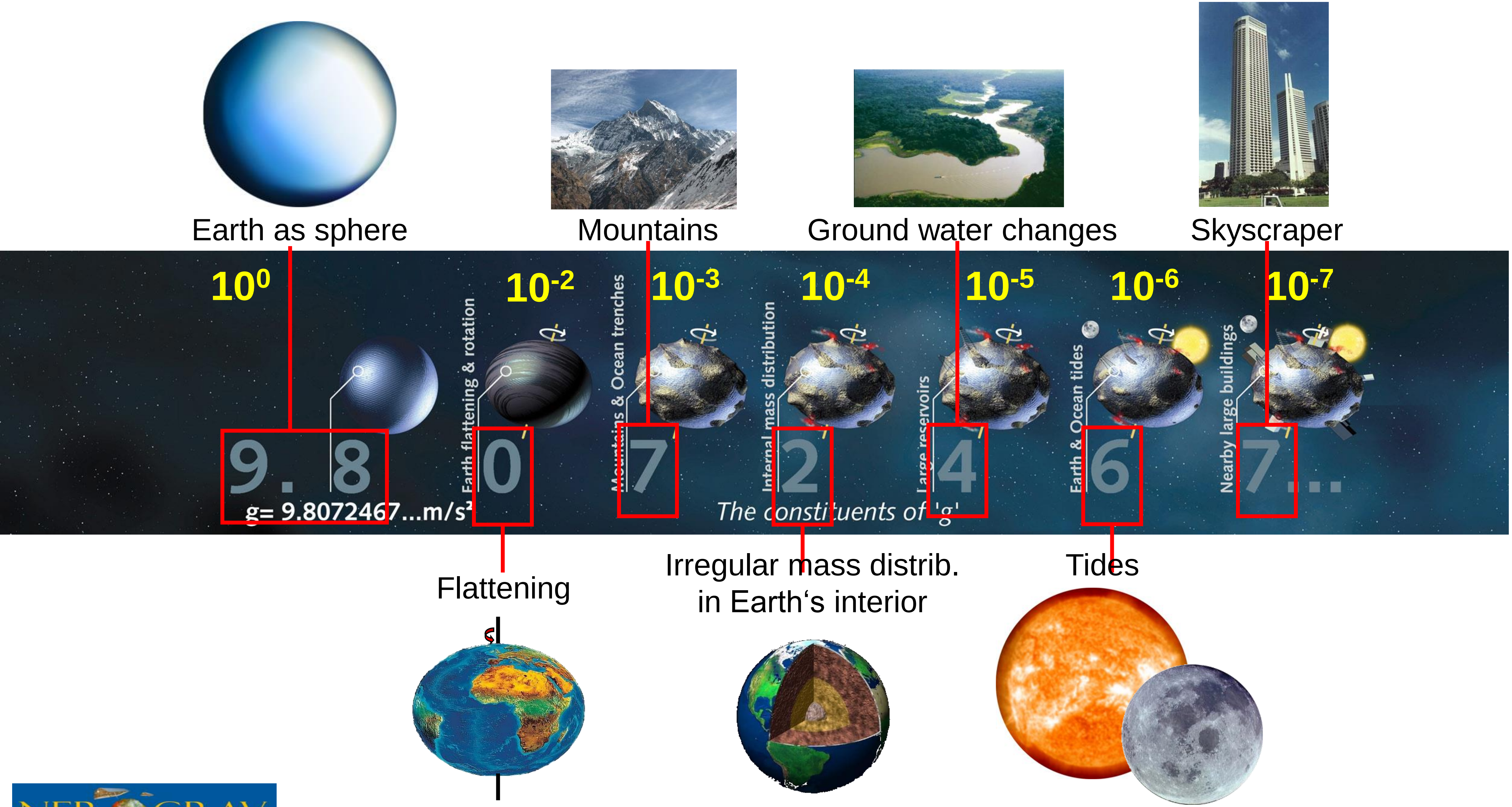


Temporal gravity variations

– Long-wavelength

– Weekly to monthly

1. Mass and Gravity



2. Gravity Observing Techniques – Gravimetry & Altimetry

➢ Terrestrial data bases

– Heterogeneous data distribution

– Heterogeneous accuracy

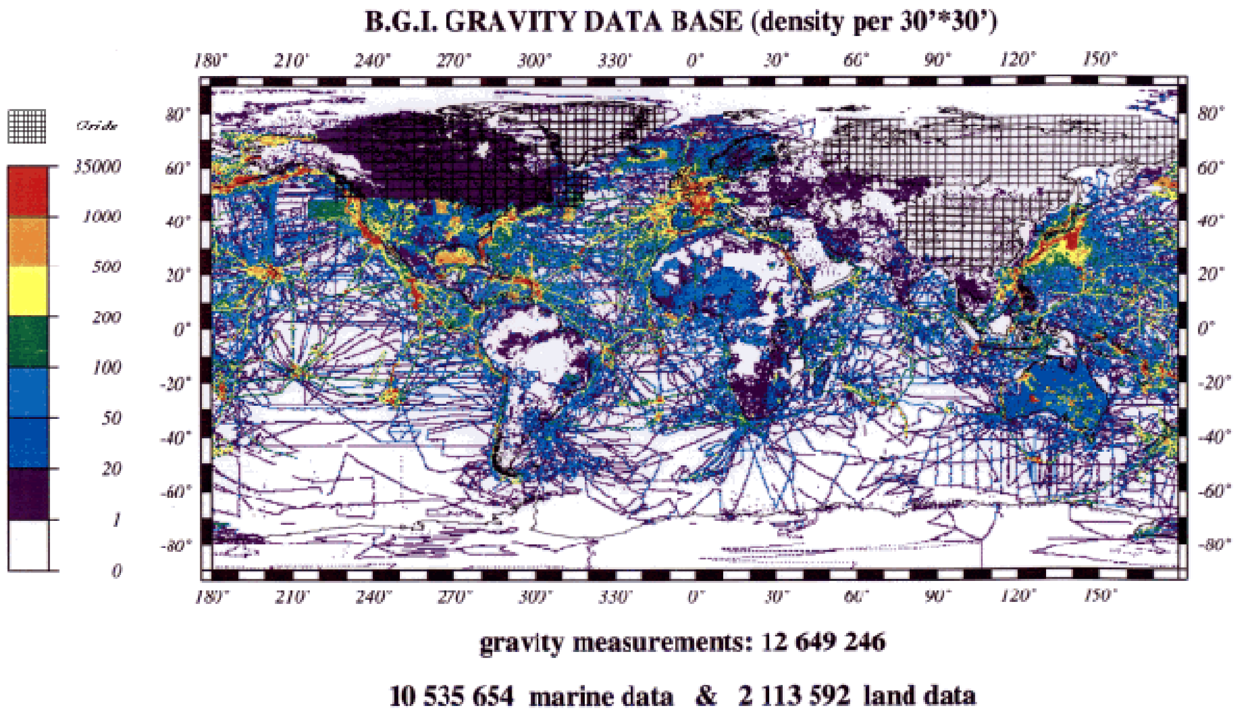
– Contains also high-frequency signal

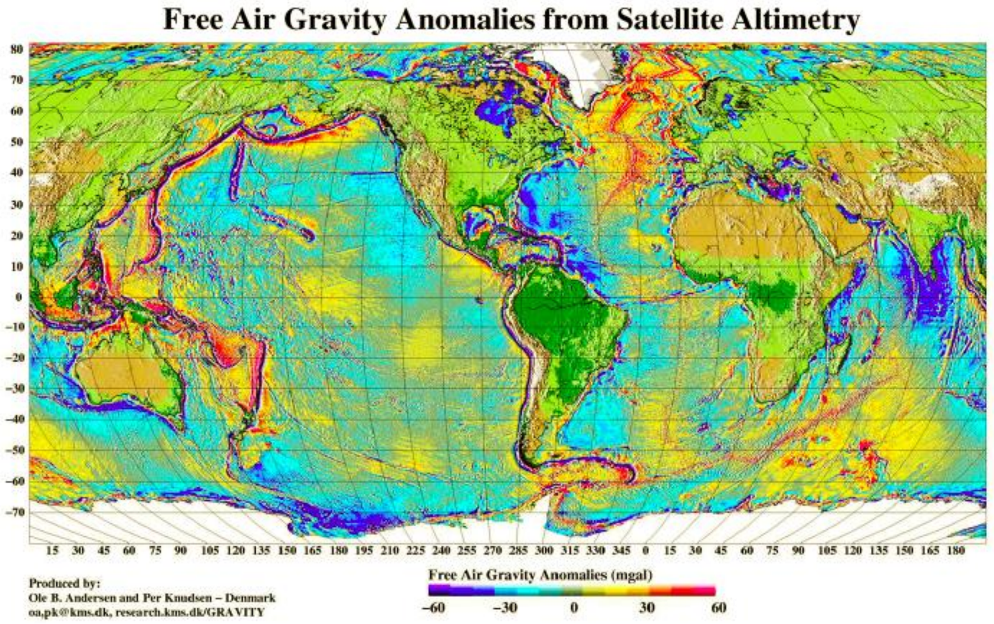
➢ Altimetric gravity

– Indirect method to derive gravity from Mean Sea surface with MDT corrections

– Covers oceans (problem: coastal areas)

– Contains also high-frequency signal



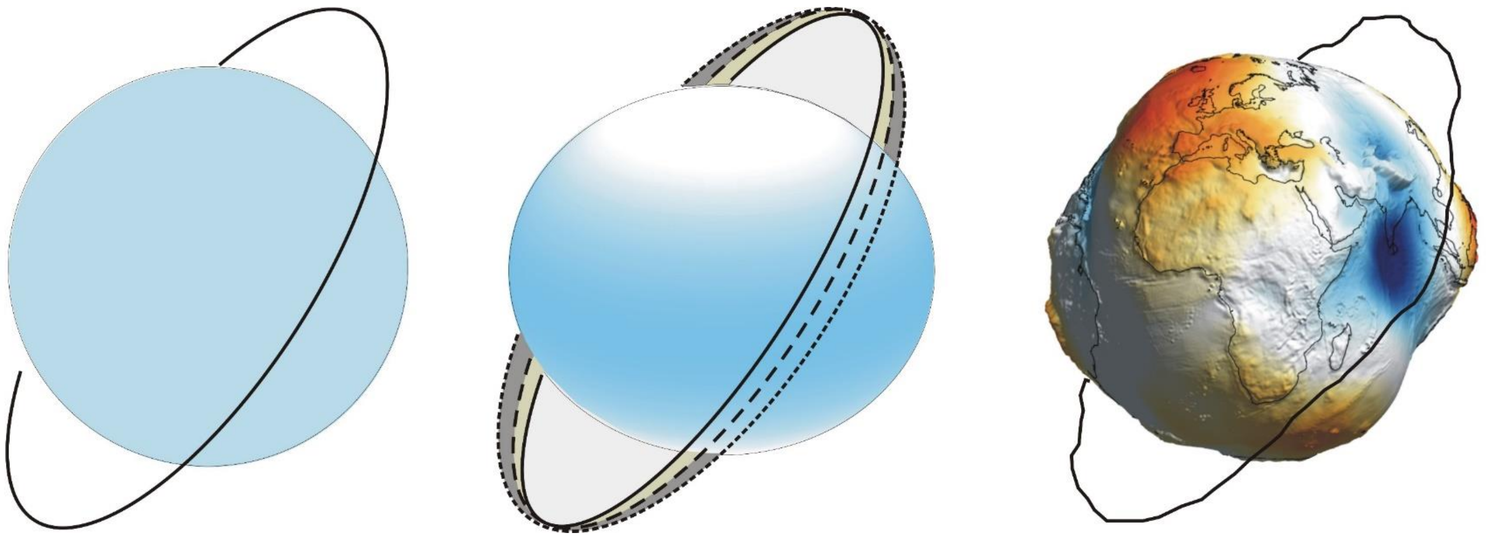


2. Gravity Observing Techniques - Satellites

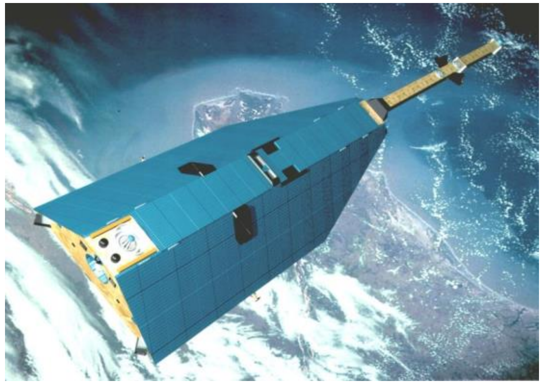
➢ Gravity satellites

Gravity from:

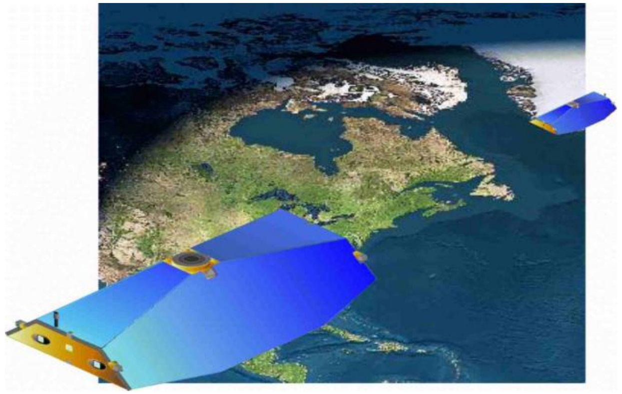
- satellite orbits
- satellite orbit differences
- acceleration differences (direct gravity functional)



SLR



CHAMP



GRACE / GRACE-FO

GOCE

2. Gravity Observing Techniques - Satellites

➢ **How to observe the Earth gravity field from space in a free-fall experiment (satellite)?**

Equation of motion

The equation of motion is composed of the ***central term*** (not disturbed) and the ***terms for all disturbing forces (F_S).*** The central term is the central force of the Earth gravity field.

$$\ddot{r} = -\frac{GM}{R^3} r + F_S$$

Observable = Acceleration
By what instrument on satellite?

$$\dot{r} = \int \left(-\frac{GM}{R^3} r + F_S \right) dt$$

Observable = Velocity
Range rates between satellites

$$r = \iint \left(-\frac{GM}{R^3} r + F_S \right) dt^2$$

Observable = Position
GNSS or range between satellites

F_S is composed by:

- Earth gravity field
- Potential of the atmosphere
- Lunar gravity field
- Gravity fields of sun and planets
- Earth tide potential
- Ocean tides potential
- Air drag
- Solar pressure
- Earth albedo

If impact of Earth gravity field on observables shall be quantified all other forces need to be known with sufficient accuracy.

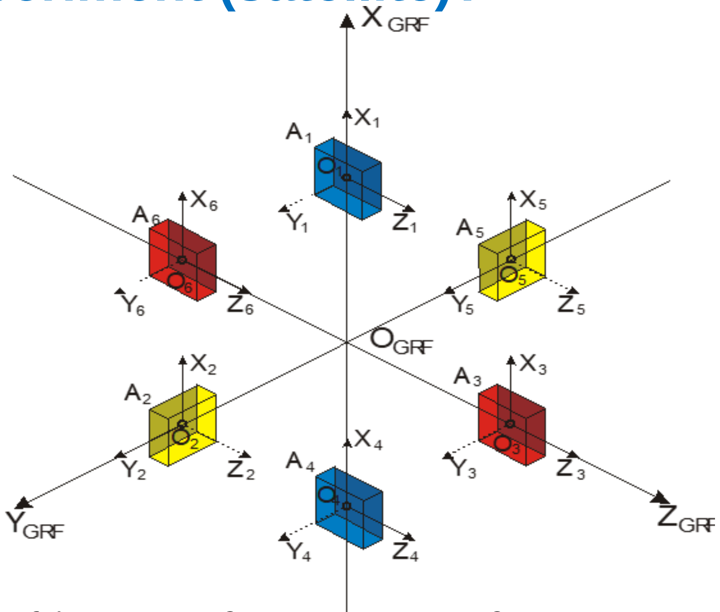
2. Gravity Observing Techniques - Satellites

➢ How to observe the Earth gravity field from space in a free-fall experiment (satellite)?

Accelerometers on Satellites

$$\underline{a} = -\underline{\underline{V}}_{ij} \cdot \underline{r} + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

the linear acceleration by the gravity potential gradients	the linear acceleration by satellite angular accelerations	the centrifugal acceleration by satellite angular rotation



$\underline{r}$: Vector from center of mass to accelerometer

Observable = Acceleration

Gravity gradiometer composed by a set of accelerometers

Gravity gradient along a baseline computed from acceleration differences between two accelerometers along the baseline and correction (observation) of rotational accelerations.

More than 1 answer might be correct !!!

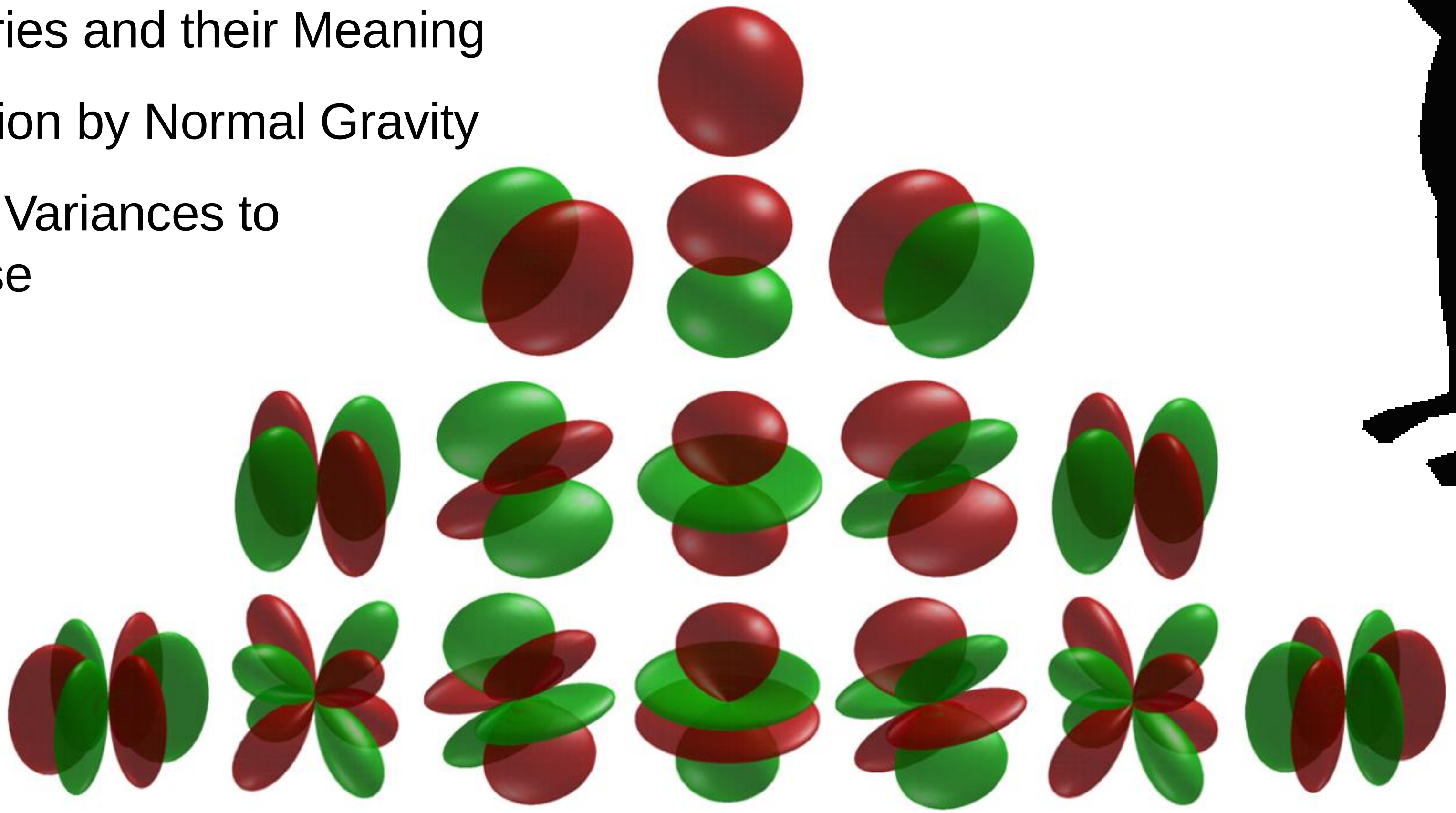
Please write down the letter of the correct answer(s).

Which observation type contains the highest high-frequency gravity information?

- C: satellite altimetry
- G: terrestrial gravity
- F: satellite gravity: GRACE
- H: satellite gravity: GOCE

How can we describe the global gravity field mathematically?

3. Spherical Harmonics Series and their Meaning
4. Gravity Field Approximation by Normal Gravity
5. Signal and Error Degree Variances to describe Signal and Noise

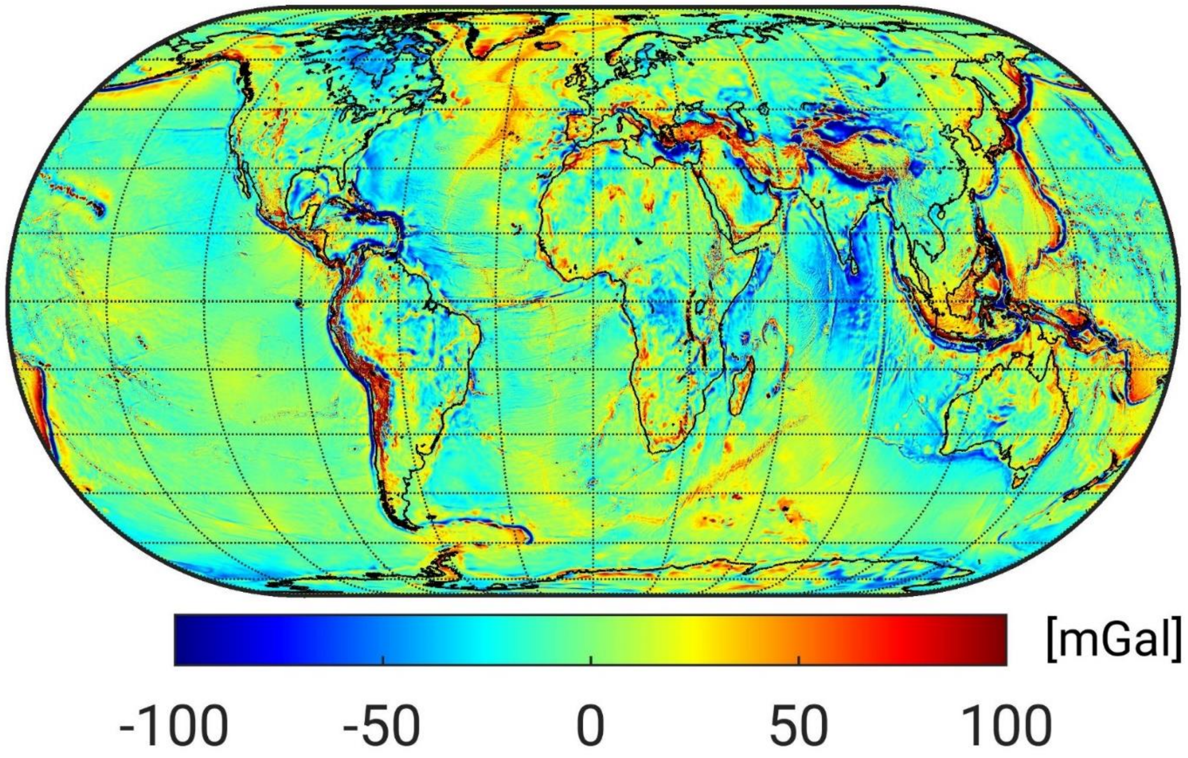


3. Spherical Harmonic Series Expansion as Solution of Laplace Equation

Laplace Equation: $div\ grad\ V = \nabla \cdot \nabla\ V = \Delta V = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) = 0$

$$V(r,\theta,\lambda) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\cdot\left[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right]$$

<u>Definition</u>: A function for which $\Delta V = 0$ is fulfilled, is called HARMONIC.

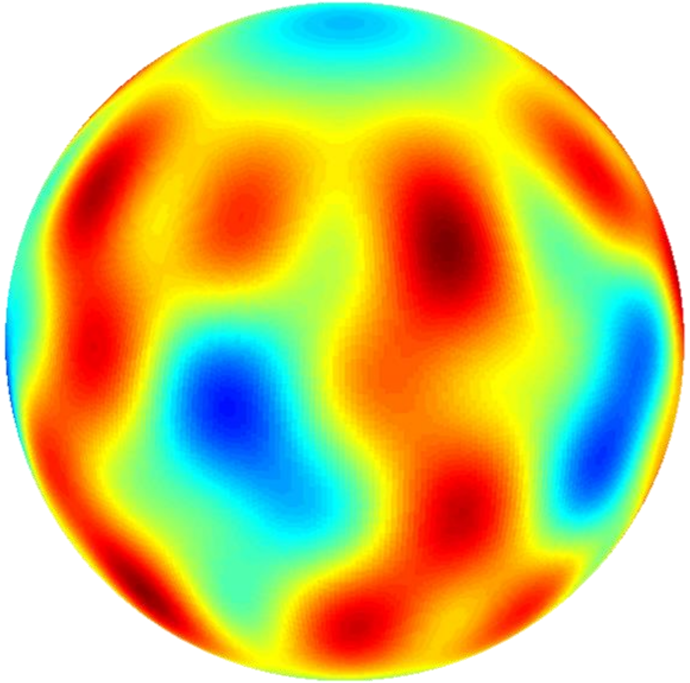


3. Surface Spherical Harmonics (SHs)

$$\begin{Bmatrix} R_{nm}(\theta,\lambda) \\ S_{nm}(\theta,\lambda) \end{Bmatrix} = P_{nm}(\cos\theta) \cdot \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix}$$

$$f(\theta,\lambda) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left[A_{nm} \cdot R_{nm}(\theta,\lambda) + B_{nm} \cdot S_{nm}(\theta,\lambda) \right]$$

A_{nm}, B_{nm} … weighting coefficients



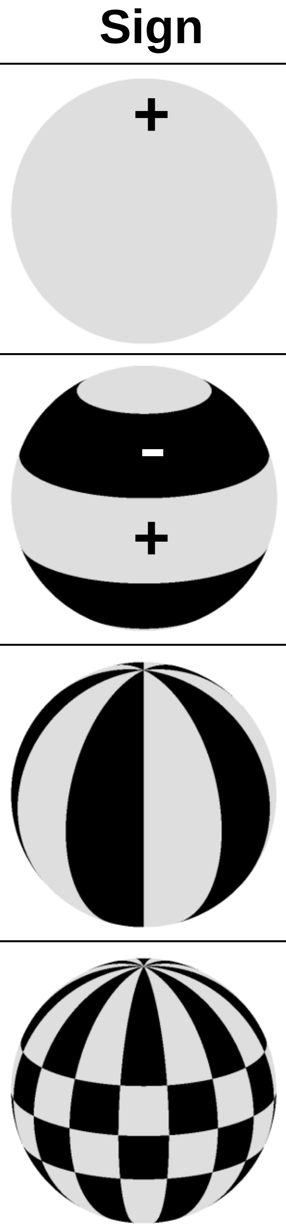
How can we generate such a function on the sphere?

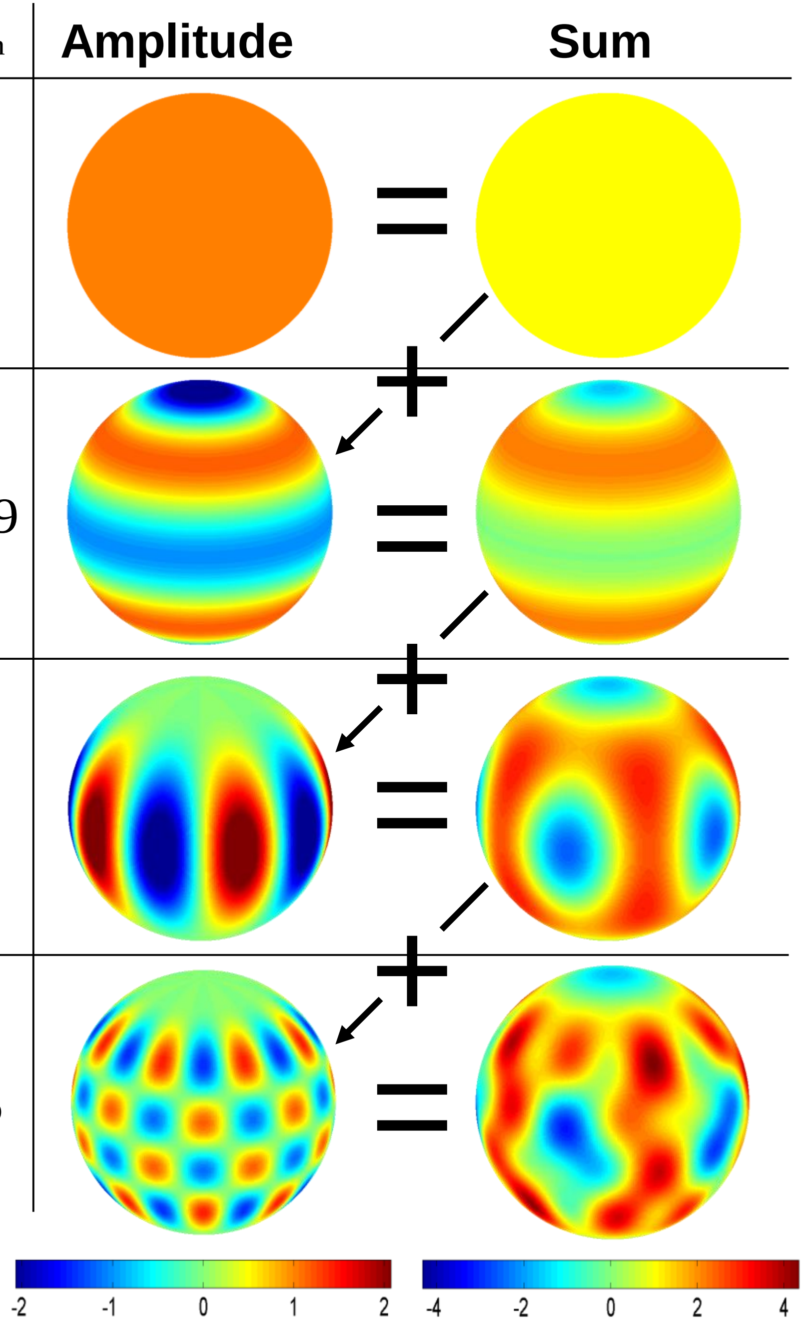
3. Surface SHs

$$f(\theta,\lambda) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{nm} \cdot R_{nm}(\theta,\lambda)$$

$A_{nm} \cdot R_{nm}(\theta,\lambda)$ $\quad \sum\sum A_{nm} \cdot R_{nm}(\theta,\lambda)$

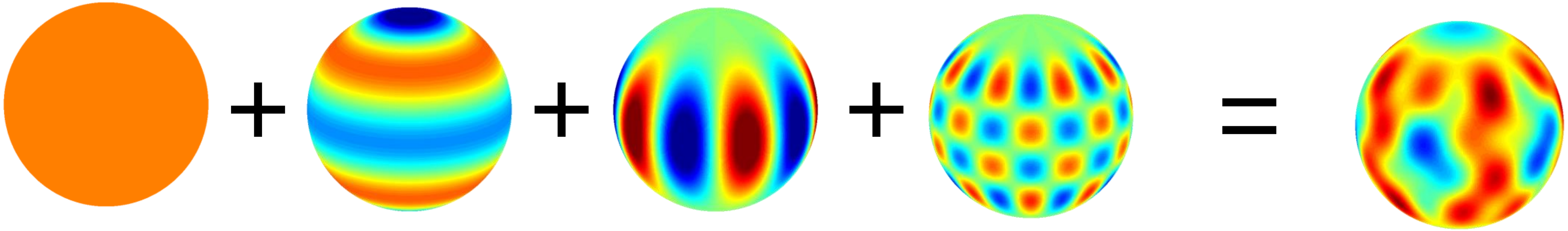
n	m	$R_{nm}(\theta,\lambda)$	A_{nm}
0	0	$\bar{P}_{00} \cdot \cos(0 \cdot \lambda)$ **mean**	1.0
4	0	$\bar{P}_{40} \cdot \cos(0 \cdot \lambda)$ **zonal**	-0.9
5	5	$\bar{P}_{55} \cdot \cos(5 \cdot \lambda)$ **sectorial**	1.1
11	8	$\bar{P}_{11,8} \cdot \cos(8 \cdot \lambda)$ **tesseral**	0.6





3. Surface SHs

Our example:

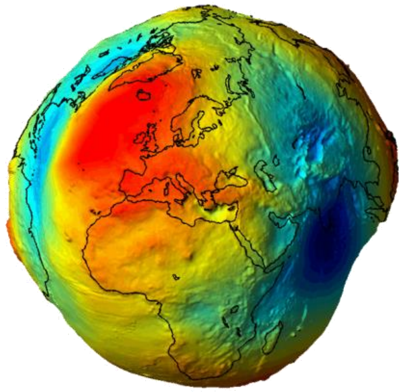


$$f(\theta,\lambda) = A_{00} \cdot R_{00}(\theta,\lambda) \quad A_{40} \cdot R_{40}(\theta,\lambda) \quad A_{55} \cdot R_{55}(\theta,\lambda) \quad A_{11,8} \cdot R_{11,8}(\theta,\lambda)$$

Reality

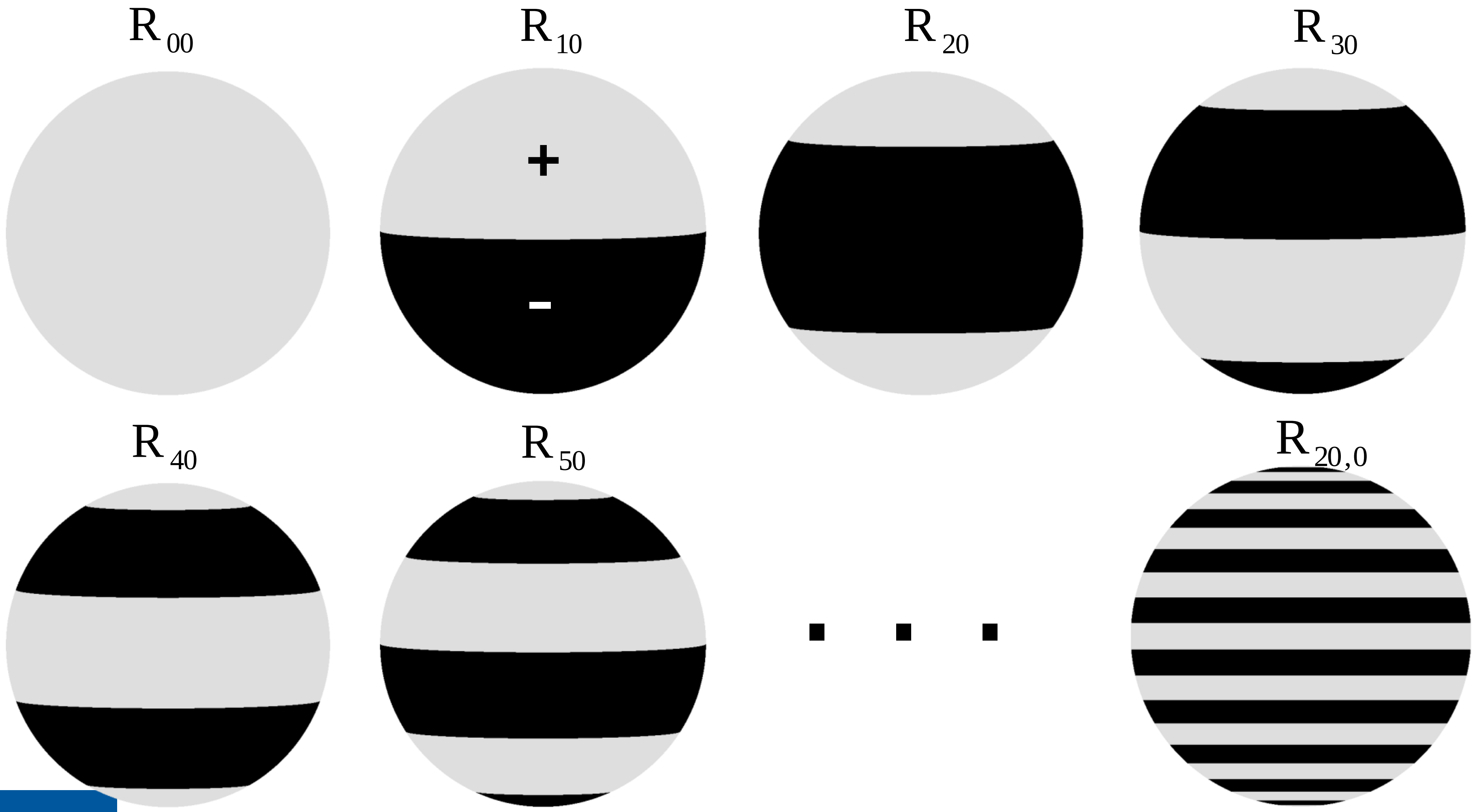
Modell coefficients

$$V(\theta,\lambda) = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} A_{nm} R_{nm}(\theta,\lambda) + B_{nm} S_{nm}(\theta,\lambda)$$



3. Zonal Base Functions: $m = 0$

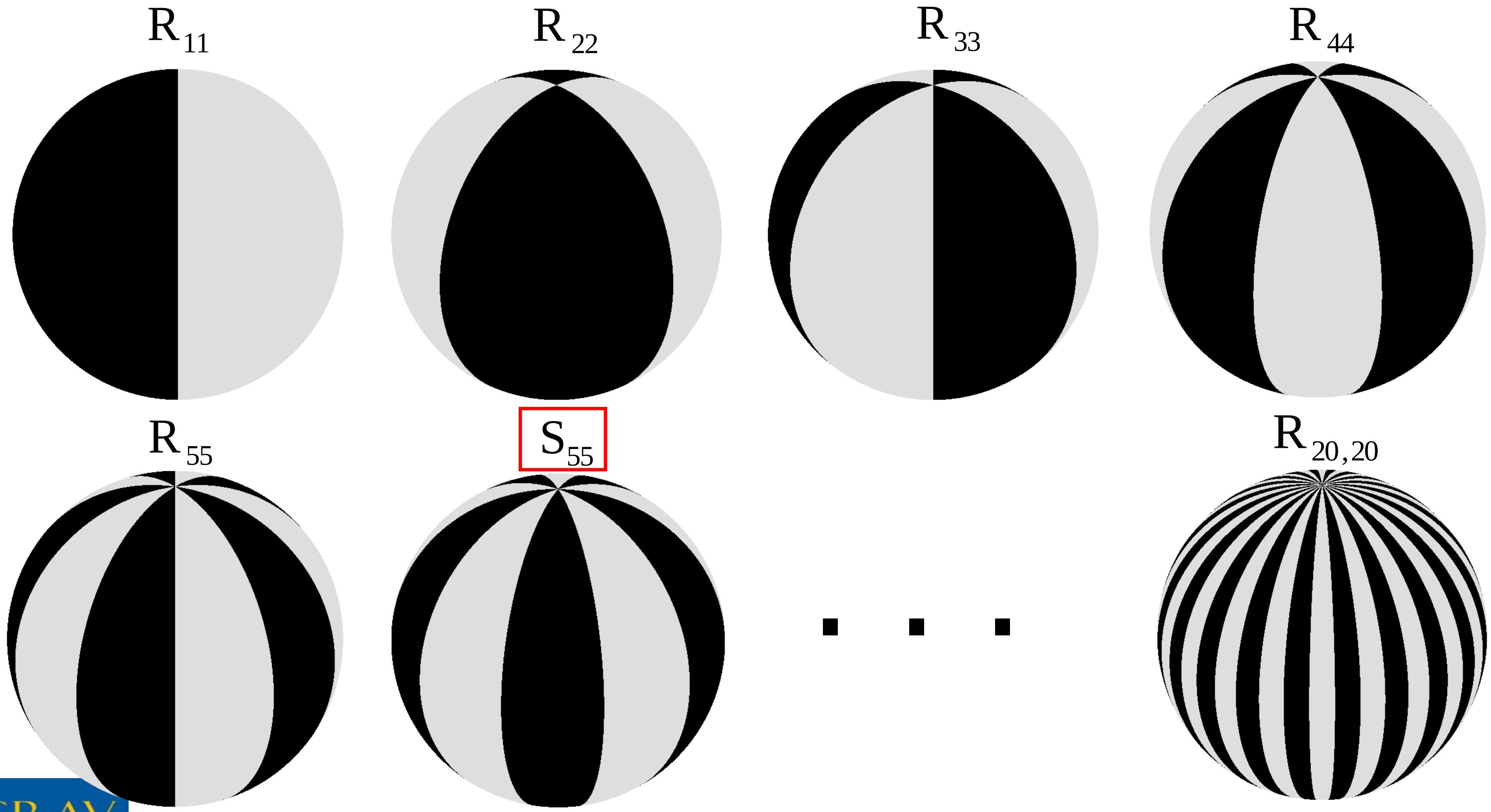
$R_{n0} = P_{n0} \cdot \cos(0 \cdot \lambda) = P_{n0}$	North pole: +1	n zeros in θ (latitude)
$S_{n0} = P_{n0} \cdot \sin(0 \cdot \lambda) = 0$	South pole: +/–1	no zeros in λ (longitude)



zonals:
$m = 0$

3. Sectorial Base Functions: $m = n$

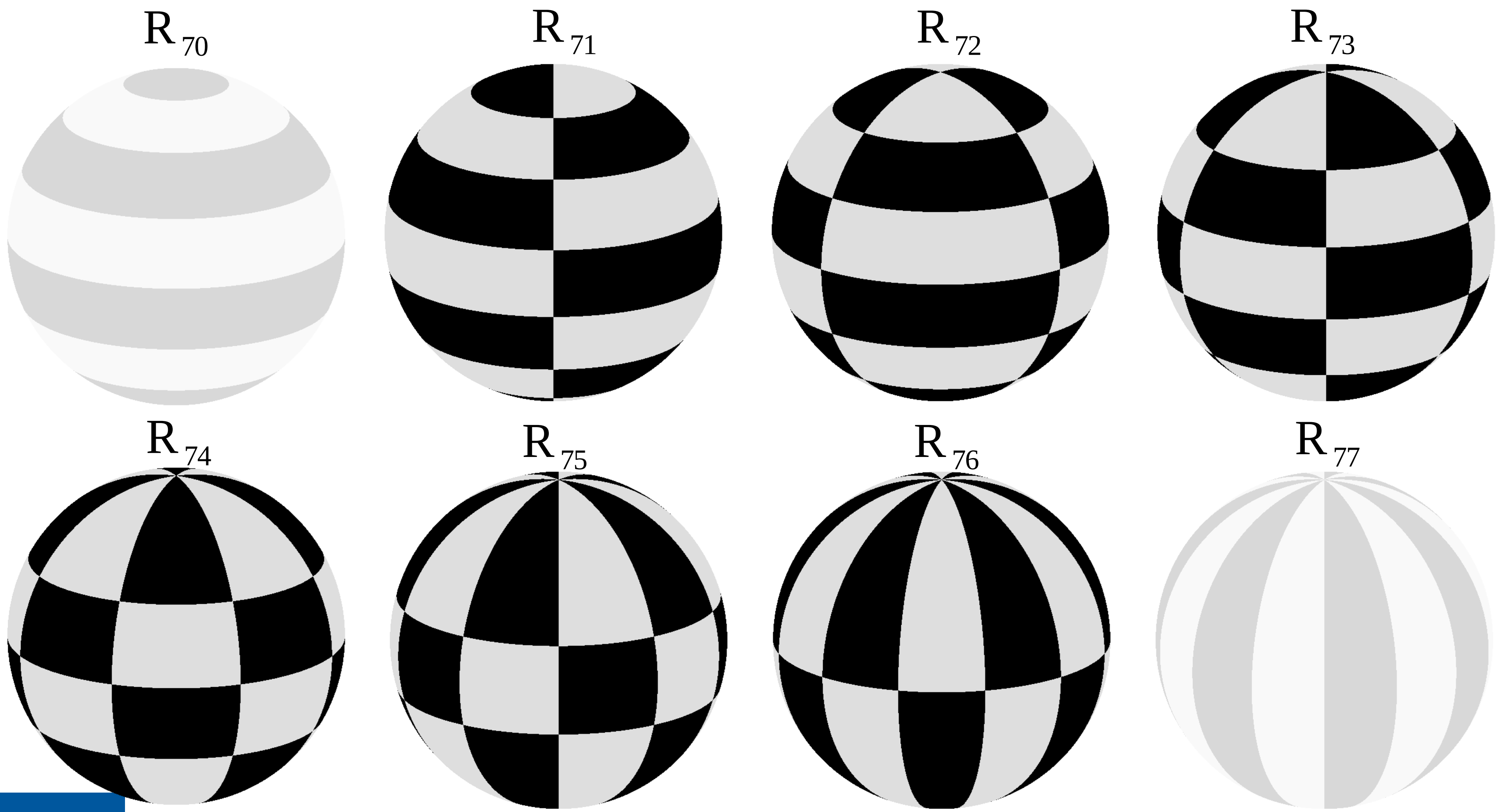
$\begin{Bmatrix} R_{nn} \\ S_{nn} \end{Bmatrix} = P_{nn} \cdot \begin{Bmatrix} \cos(n\lambda) \\ \sin(n\lambda) \end{Bmatrix}$	North pole: 0 South pole: 0	no zeros in θ (latitude) $2m$ zeros in λ (longitude)



sectorials:
$m = n$

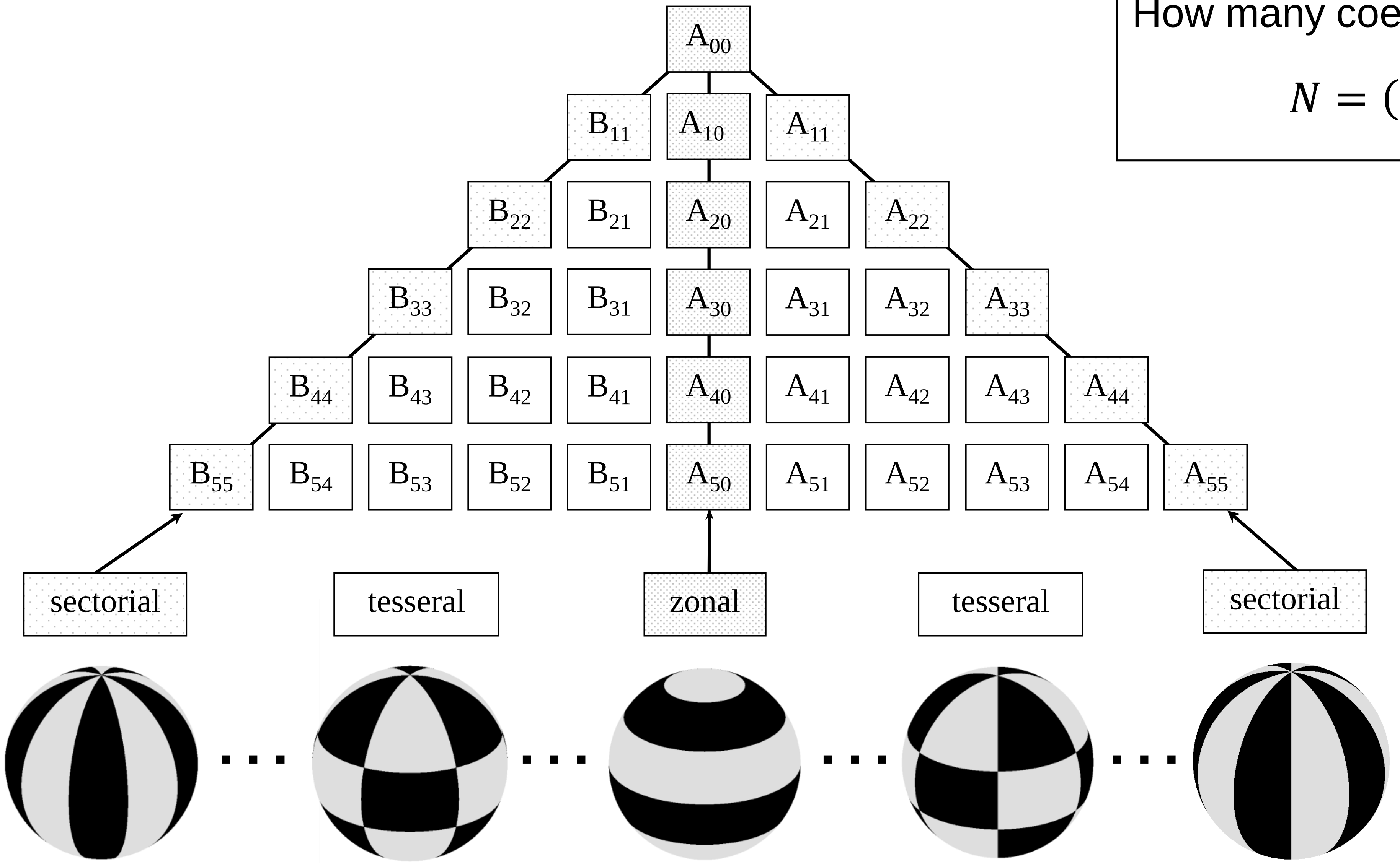
3. Tesseral Base Functions

$\begin{Bmatrix} R_{nm} \\ S_{nm} \end{Bmatrix} = P_{nm} \cdot \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix}$	North pole: 0 South pole: 0	$n-m$ (+2) zeros in θ (latitude) $2m$ zeros in λ (longitude)



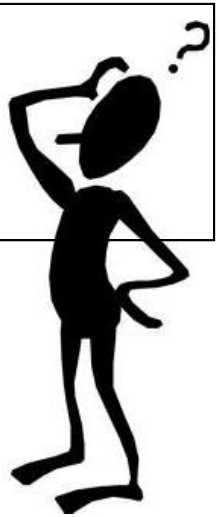
tesserals:
$m \neq n, m \neq 0$

3. SH Base Functions: Triangle Representation



How many coefficients do exist?

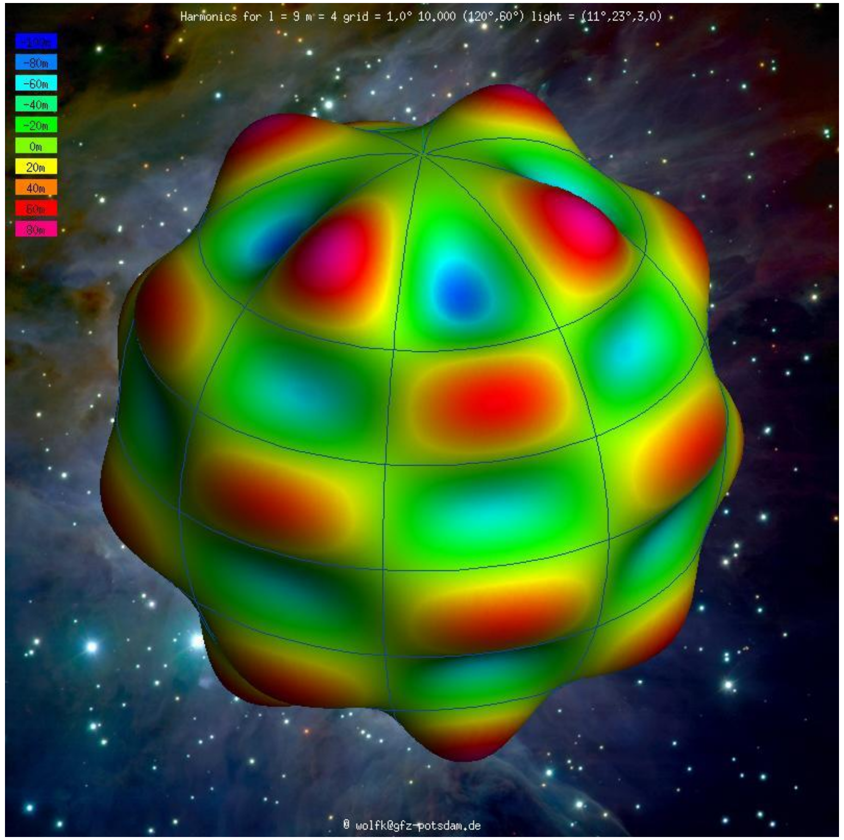
$$N = (n_{max} + 1)^2$$



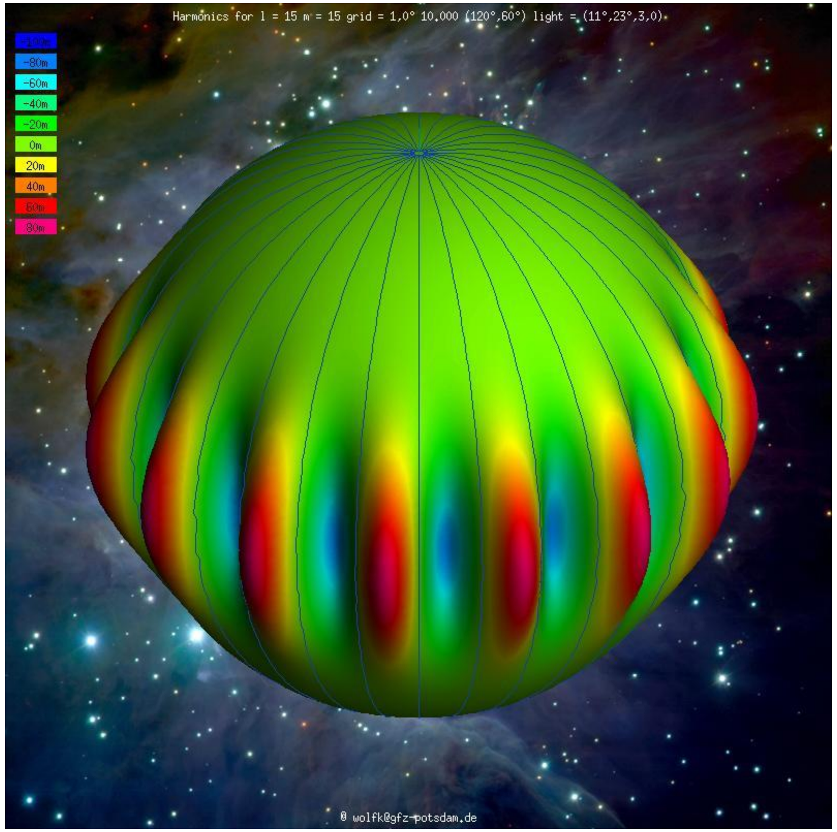
3. SH Base Functions: Visualisation

3D visualisation of surface SHs

http://icgem.gfz-potsdam.de/vis3d/tutorial



R_{94}



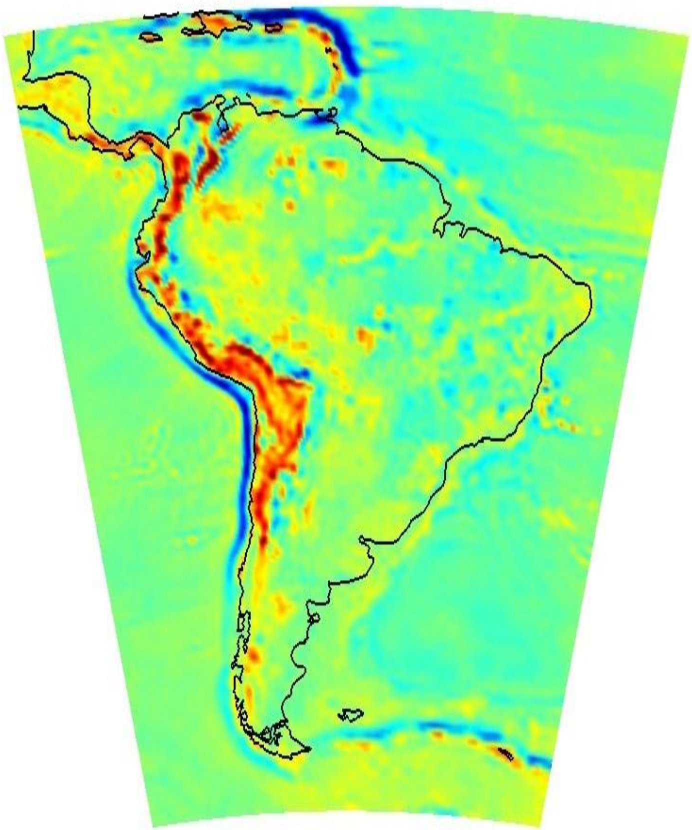
$R_{15,15}$

3. Harmonic Degree and Spatial Wavelength

$$f(\theta,\lambda) = \sum_{n=0}^{n_{max}} \sum_{m=0}^{n} P_{nm}(\cos\theta)\,[A_{nm}\cos(m\lambda) + B_{nm}\sin(m\lambda)]$$

f … gravity anomalies (in region South America)

n_{max}	# coeff.	λ [km]
20	441	1000
50	2601	400
100	10 201	200
250	63 001	80



λ … spatial (half) wavelength

$$\lambda[\mathrm{km}] = \frac{20\,000}{n_{max}}$$

3. Harmonic Degree and Spatial Wavelength

Gravity anomalies
(in region South America)

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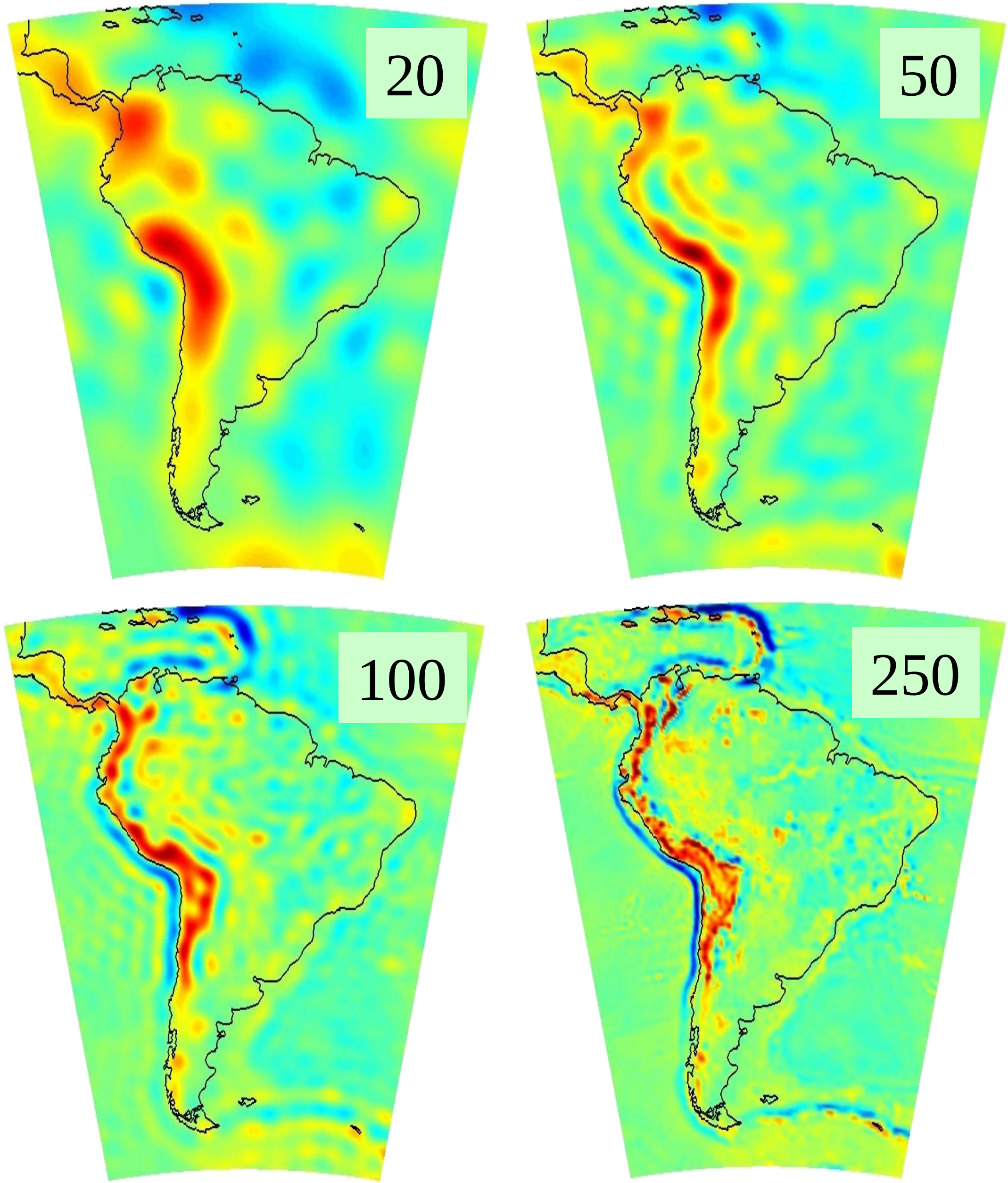
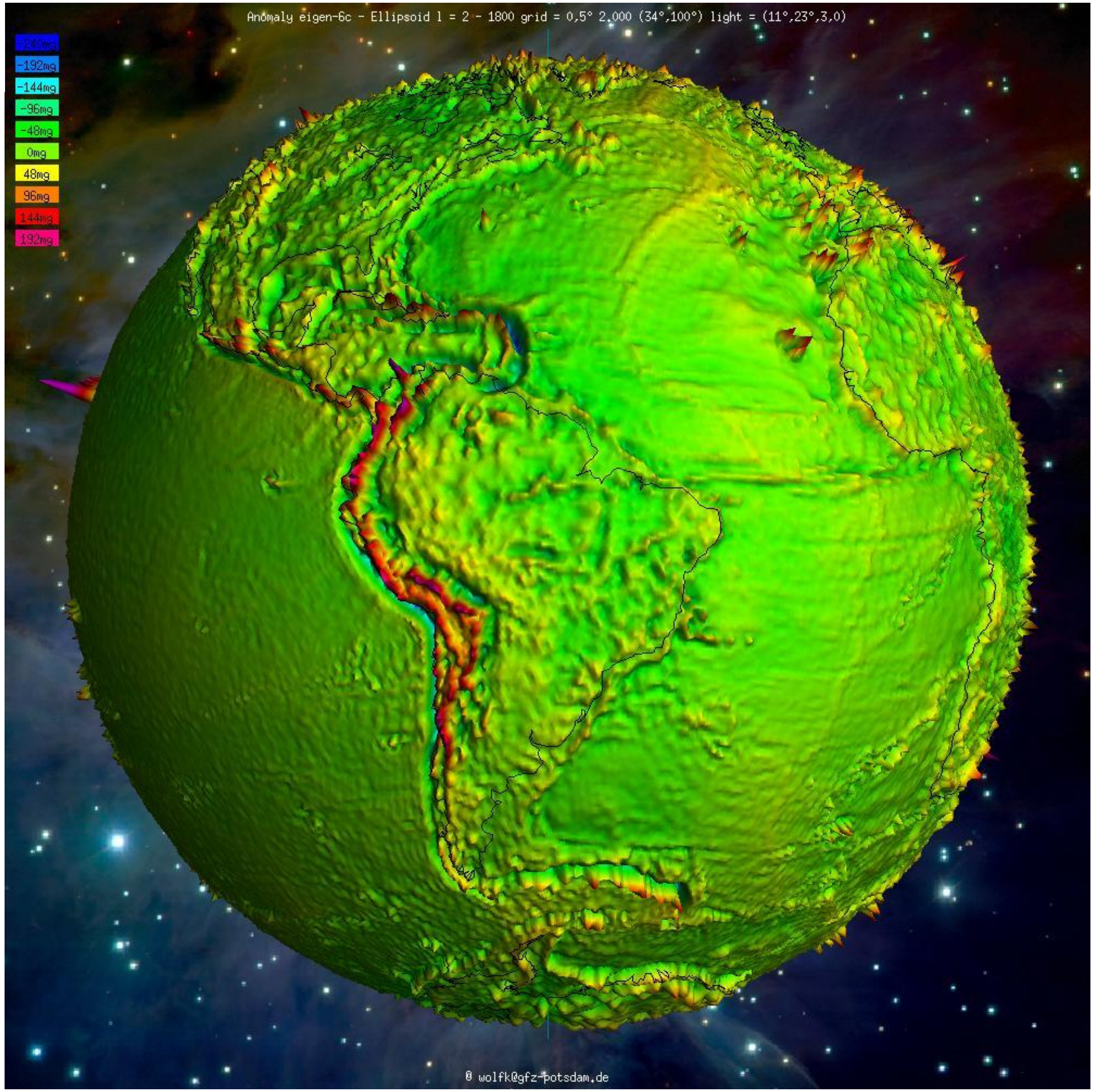


3. Harmonic Degree and Spatial Wavelength: Visualization

n_{max} = 1800



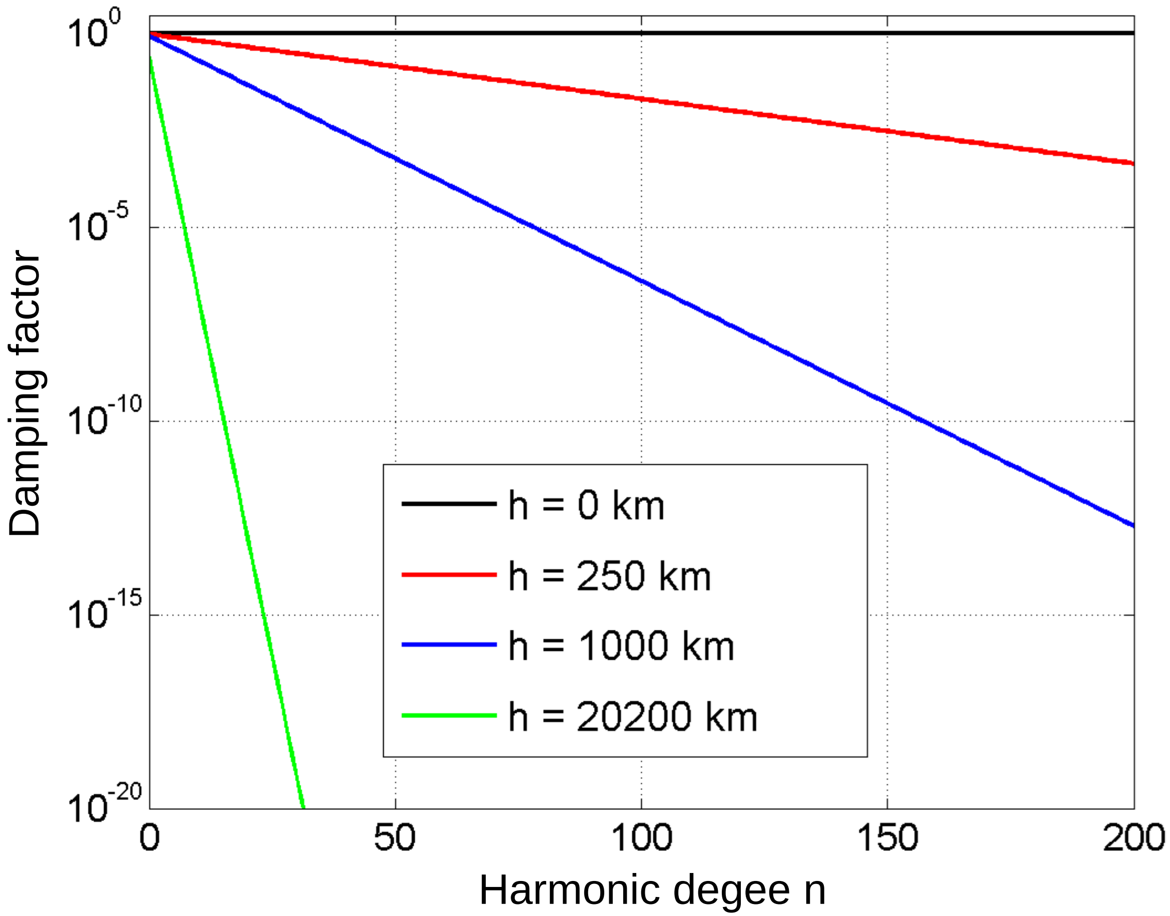
http://icgem.gfz-potsdam.de/vis3d/tutorial

3. Height Dependence

$$V(r,\theta,\lambda) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\left[\bar{C}_{nm}\cos(m\lambda)+\bar{S}_{nm}\sin(m\lambda)\right]$$

$$r = R + h$$

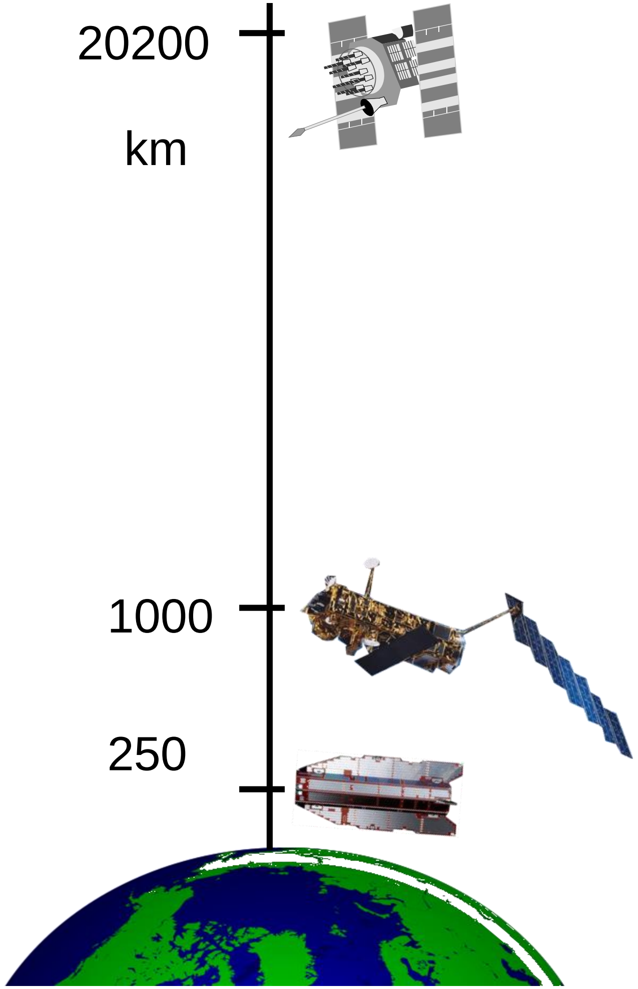
$\left(\frac{R}{r}\right)^{n+1}$ for different heights h

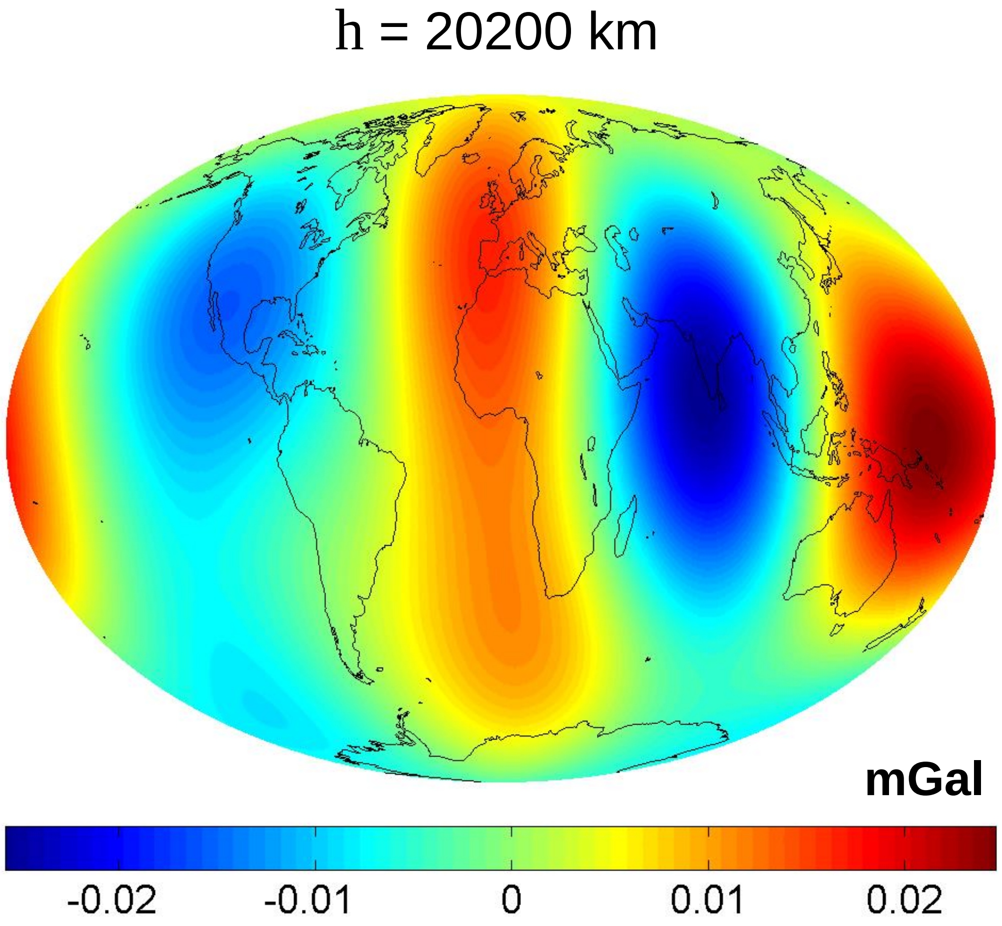


→ The higher the degree n, the stronger is signal attenuation

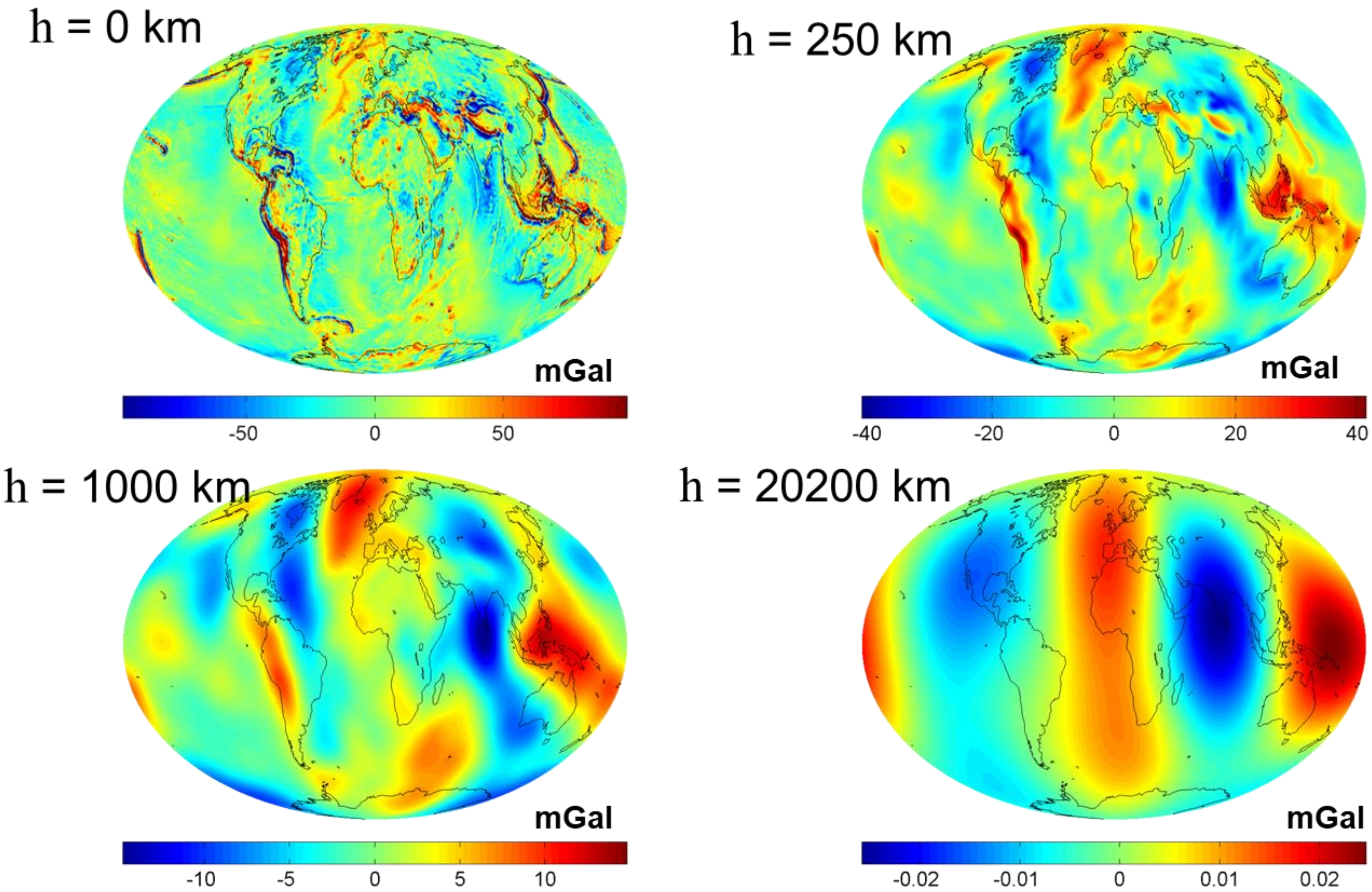
3. Height Dependence

Gravity field acceleration in
dependence of height h





3. Gravity Anomaly in Dependence of Height



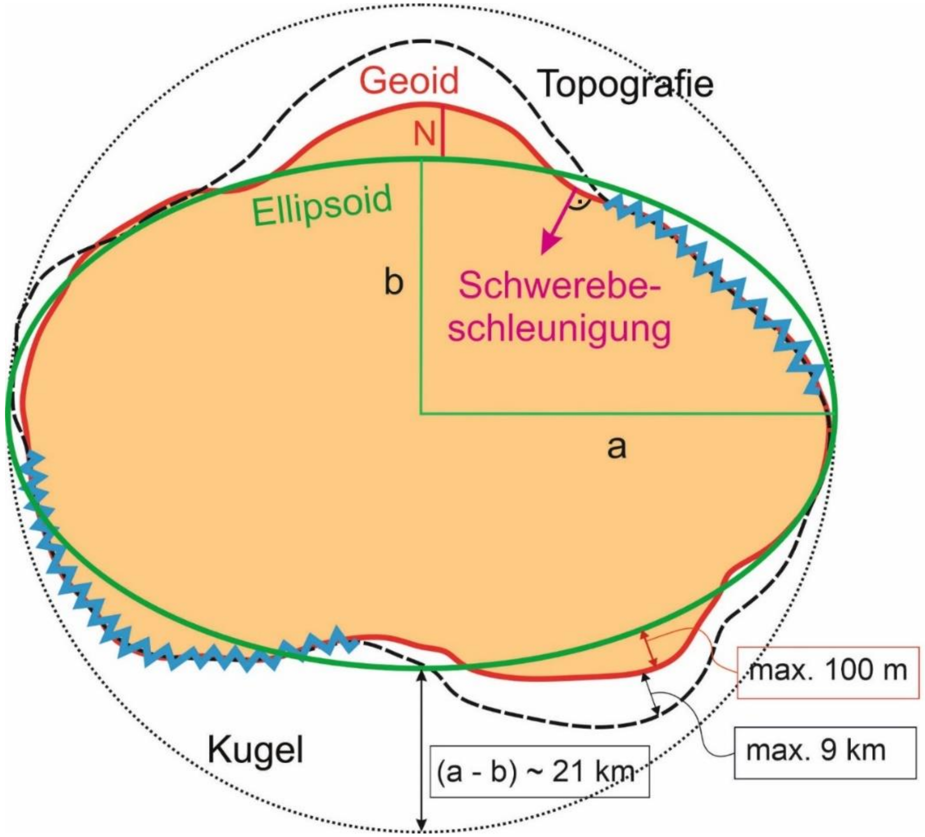
4. Normal Gravity

Gravity observable:

$$b = b\,(S, W)$$

Location of obs. (e.g. Earth surface, satellite altitude) → S

Gravity potential → W



- Central task of physical geodesy: determination of W (and S) from b
- b is usually connected in a non-linear way with W and S

  → linearization required

  → Approximate solution as Taylor point

  → Normal potential $U \approx W$

4. SH Expansion of Normal Gravity

Normal potential: $\boxed{U = V' + Z}$

$$V'(r,\theta,\lambda) = \frac{GM_0}{a} \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_{nm}(\cos\theta)\,[c_{nm}\cos(m\lambda) + s_{nm}\sin(m\lambda)]$$

Gravitational potential

$$Z(r,\theta,\lambda) = \frac{1}{2}\omega^2 r^2 sin^2\theta$$

Centrifugal potential

$$\rightarrow \boxed{V'(r,\theta,\lambda) = \frac{GM_0}{a} \sum_{n=0\,(2)}^{\infty} \left(\frac{a}{r}\right)^{n+1} P_{n0}(\cos\theta)\, c_{n0}}$$

- rotational symmetry → no dependence on λ → only zonals $m = 0$
- symmetry w.r.t. equator → only even zonals

In practice: $\infty \rightarrow$ 8 or 10, because fast decrease in amplitude

Disturbing potential: $T = W - U = V + Z - (V' + Z) = V - V'$

4. Global Gravity Model (ICGEM format)

EGM 2008

```
product_type                gravity_field
modelname                   EGM2008
earth_gravity_constant      0.3986004415E+15
radius                      0.63781363E+07
max_degree                  2190
errors                      calibrated
norm                        fully_normalized
tide_system                 tide_free

url                   http://earth-info.nima.mil/GandG/

key      L    M             C                        S                     sigma C             sigma S
end_of_head =====================================================================================================
gfc      0    0    1.0d0                    0.0d0                    0.0d0              0.0d0
gfc      2    0   -0.484165143790815D-03    0.000000000000000D+00    0.7481239490D-11   0.0000000000D+00
gfc      2    1   -0.206615509074176D-09    0.138441389137979D-08    0.7063781502D-11   0.7348347201D-11
gfc      2    2    0.243938357328313D-05   -0.140027370385934D-05    0.7230231722D-11   0.7425816951D-11
gfc      3    0    0.957161207093473D-06    0.000000000000000D+00    0.5731430751D-11   0.0000000000D+00
gfc      3    1    0.203046201047864D-05    0.248200415856872D-06    0.5726633183D-11   0.5976692146D-11
gfc      3    2    0.904787894809528D-06   -0.619005475177618D-06    0.6374776928D-11   0.6401837794D-11
gfc      3    3    0.721321757121568D-06    0.141434926192941D-05    0.6029131793D-11   0.6028311182D-11
gfc      4    0    0.539965866638991D-06    0.000000000000000D+00    0.4431111968D-11   0.0000000000D+00
gfc      4    1   -0.536157389388867D-06   -0.473567346518086D-06    0.4568074333D-11   0.4684043490D-11
gfc      4    2    0.350501623962649D-06    0.662480026275829D-06    0.5307840320D-11   0.5186098530D-11
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gfc      5    1   -0.629211923042529D-07   -0.943698073395769D-07    0.2989077566D-11   0.3143313186D-11
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gfc      5    3   -0.451847152328843D-06   -0.214955408306046D-06    0.4725934077D-11   0.4688985442D-11
gfc      5    4   -0.295328761175629D-06    0.498070550102351D-07    0.5332198489D-11   0.5302621028D-11
gfc      5    5    0.174811795496002D-06   -0.669379935180165D-06    0.4980396595D-11   0.4981027282D-11
gfc      6    0   -0.149953927978527D-06    0.000000000000000D+00    0.2035490195D-11   0.0000000000D+00
gfc      6    1   -0.759210081892527D-07    0.265122593213647D-07    0.2085980159D-11   0.2193954647D-11
gfc      6    2    0.486488924604690D-07   -0.373789324523752D-06    0.2603949443D-11   0.2466506184D-11
gfc      6    3    0.572451611175653D-07    0.895201130010730D-08    0.3380286162D-11   0.3347204566D-11
gfc      6    4   -0.860237937191611D-07   -0.471425573429095D-06    0.4535102219D-11   0.4489428324D-11
gfc      6    5   -0.267166423703038D-06   -0.536493151500206D-06    0.5097794605D-11   0.5101153019D-11
gfc      6    6    0.947068749756882D-08   -0.237382353351005D-06    0.4731651005D-11   0.4728357086D-11
.
.
```
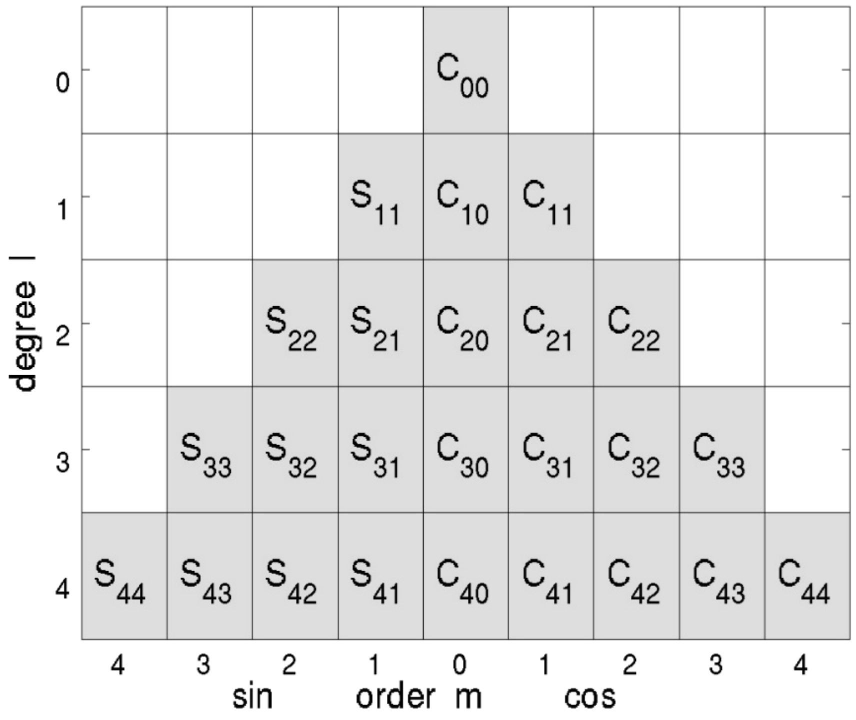
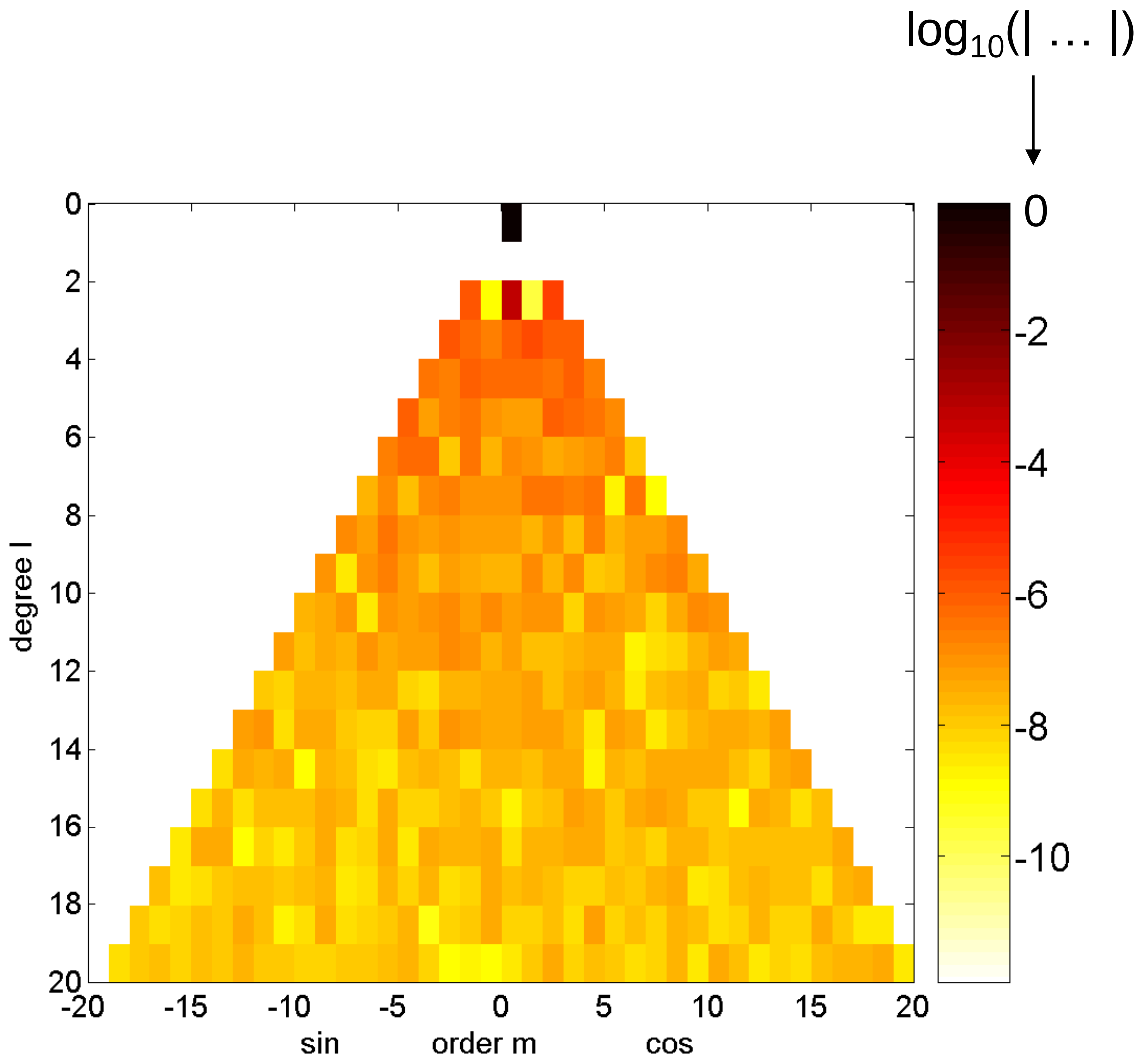

4. Global Gravity Model (ICGEM format)

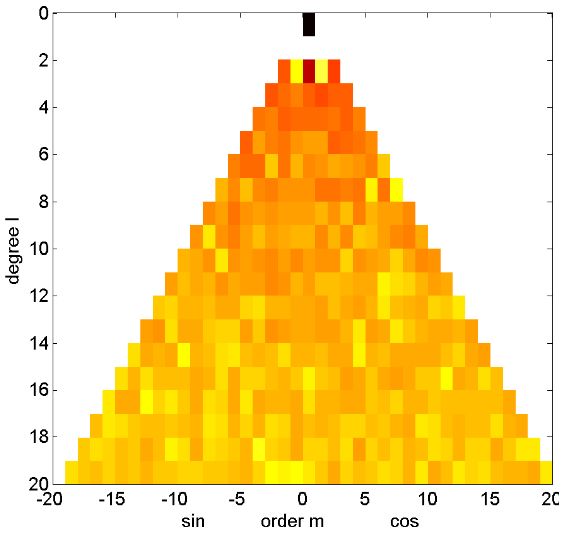


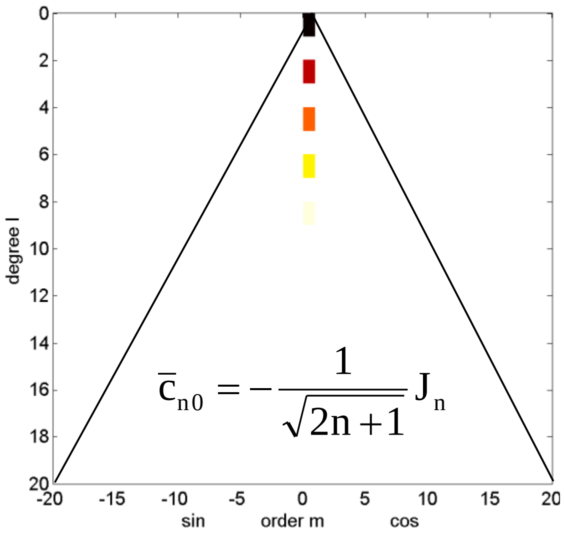
EGM 2008

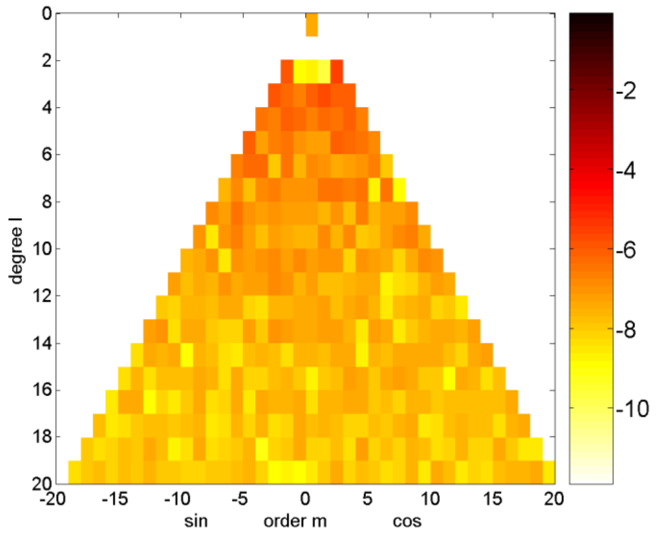


4. Disturbing Pot.

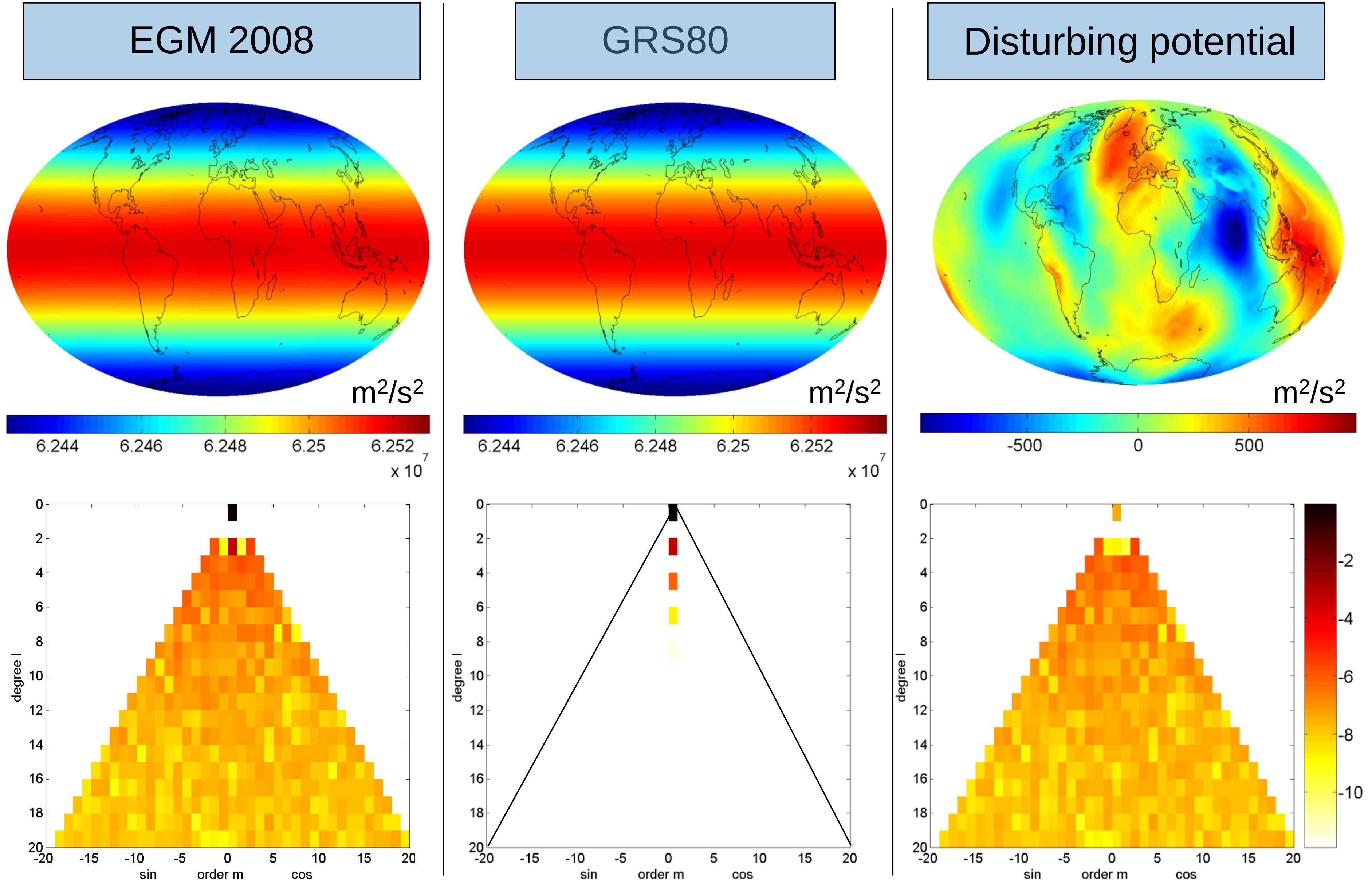
		EGM 2008 (GM = 0.3986004415D+15, R = 0.63781363D+07)		GRS80 (GM = 0.3986005D+15, a = 0.6378137D+07)		Disturbing potential	
n	m	C_nm	S_nm	C_nm	S_nm	C_nm	S_nm
0	0	1.	0.	1.	0.	-0.37013570D-07	0.
2	0	-0.48416514D-03	0.00000000D+00	-0.48416685D-03	0.	1.71110530D-09	0.
2	1	-0.20661550D-09	0.13844138D-08	0.	0.	-0.20661550D-09	0.13844138D-08
2	2	0.24393835D-05	-0.14002737D-05	0.	0.	0.24393835D-05	-0.14002737D-05
3	0	0.95716120D-06	0.	0.	0.	0.95716120D-06	0.
3	1	0.20304620D-05	0.24820041D-06	0.	0.	0.20304620D-05	0.24820041D-06
3	2	0.90478789D-06	-0.61900547D-06	0.	0.	0.90478789D-06	-0.61900547D-06
3	3	0.72132175D-06	0.14143492D-05	0.	0.	0.72132175D-06	0.14143492D-05
4	0	0.53996586D-06	0.	0.79030407D-06	0.	-0.25033820D-06	0.
4	1	-0.53615738D-06	-0.47356734D-06	0.	0.	-0.53615738D-06	-0.47356734D-06
4	2	0.35050162D-06	0.66248002D-06	0.	0.	0.35050162D-06	0.66248002D-06
4	3	0.99085676D-06	-0.20095672D-06	0.	0.	0.99085676D-06	-0.20095672D-06
4	4	-0.18851963D-06	0.30880388D-06	0.	0.	-0.18851963D-06	0.30880388D-06
6	0	-0.14995392D-06	0.	-0.16872510D-08	0.	-0.14826667D-06	0.
8	0	0.49475600D-07	0.	0.34609833D-11	0.	0.49472139D-07	0.



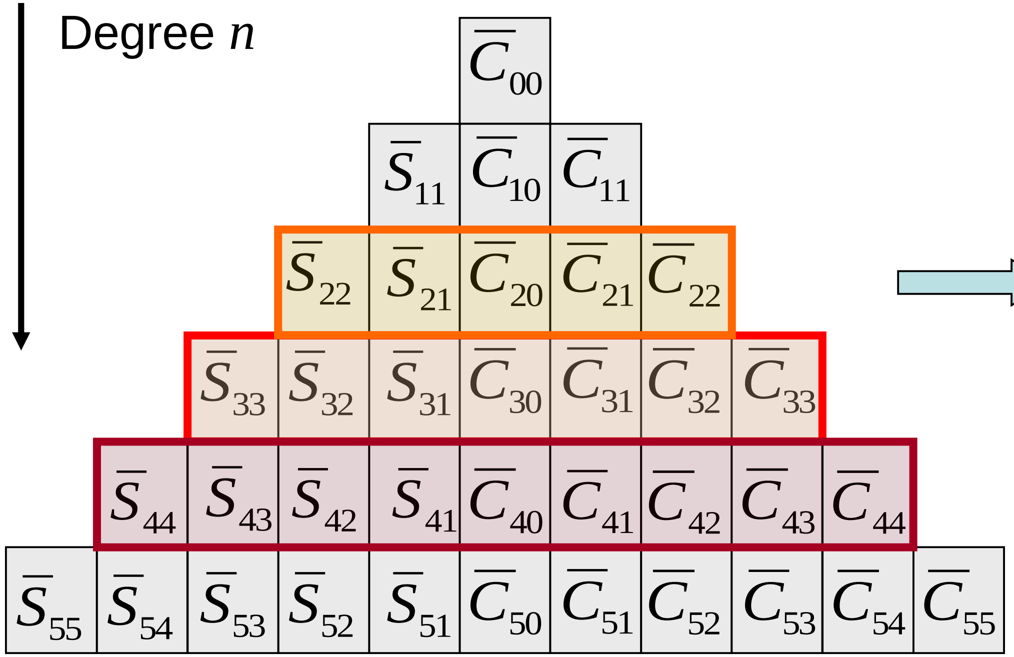




4. Disturbing Potential



5. Signal Variances and Degree Variances



Signal variances

Energy per degree n :

$$c_n = \sum_{m=0}^{n} \left(\bar{C}_{nm}^2 + \bar{S}_{nm}^2 \right)$$

Mean amplitude per degree n and per coefficient:

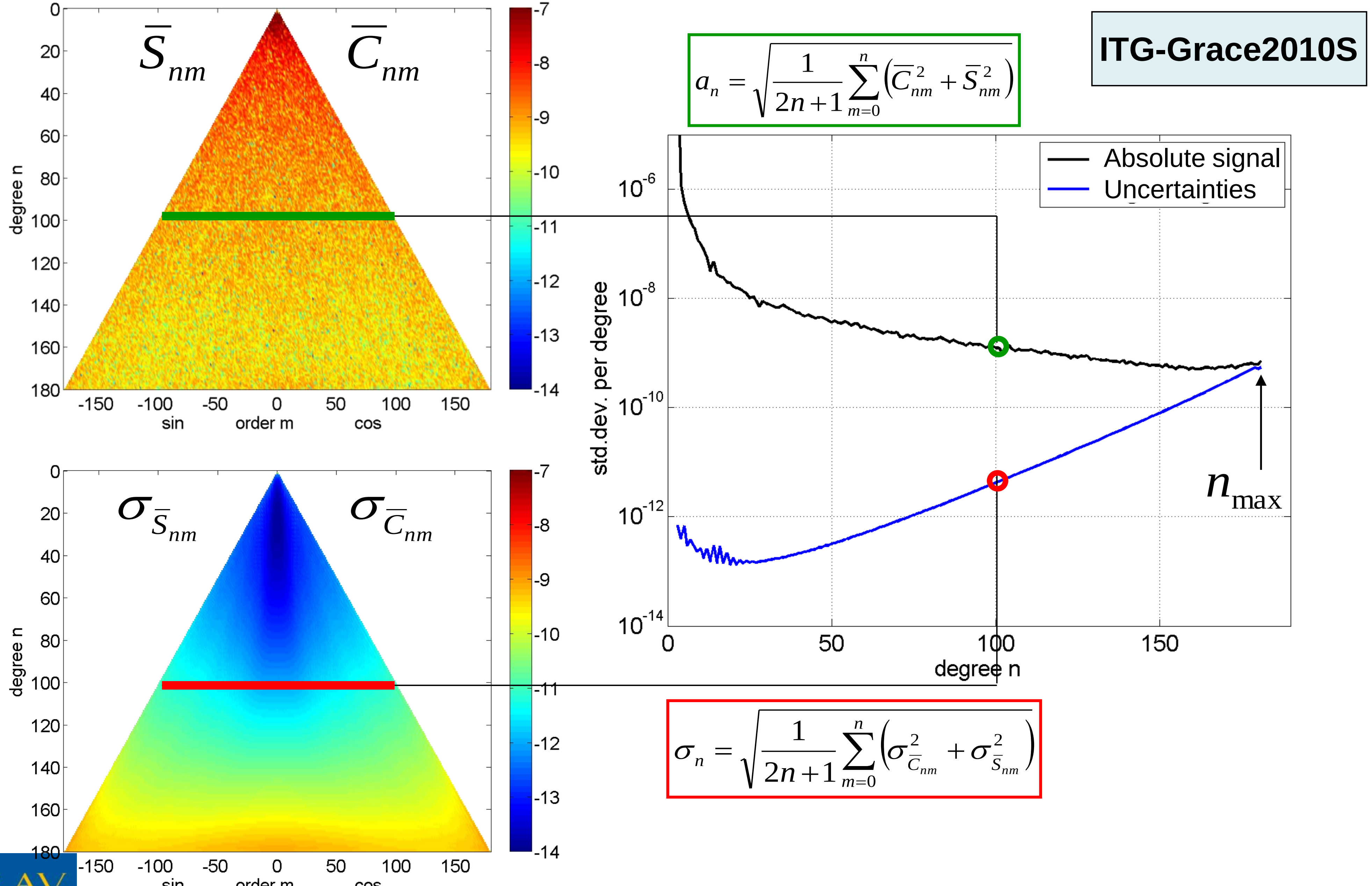
$$a_n = \sqrt{\frac{1}{2n+1} \sum_{m=0}^{n} \left(\bar{C}_{nm}^2 + \bar{S}_{nm}^2 \right)}$$

Every coefficients can be determined only with a specific accuracy:

$$\bar{C}_{nm} \pm \sigma_{\bar{C}_{nm}}; \quad \bar{S}_{nm} \pm \sigma_{\bar{S}_{nm}}$$

$$\sigma_n = \sqrt{\frac{1}{2n+1} \sum_{m=0}^{n} \left(\sigma_{\bar{C}_{nm}}^2 + \sigma_{\bar{S}_{nm}}^2 \right)}$$

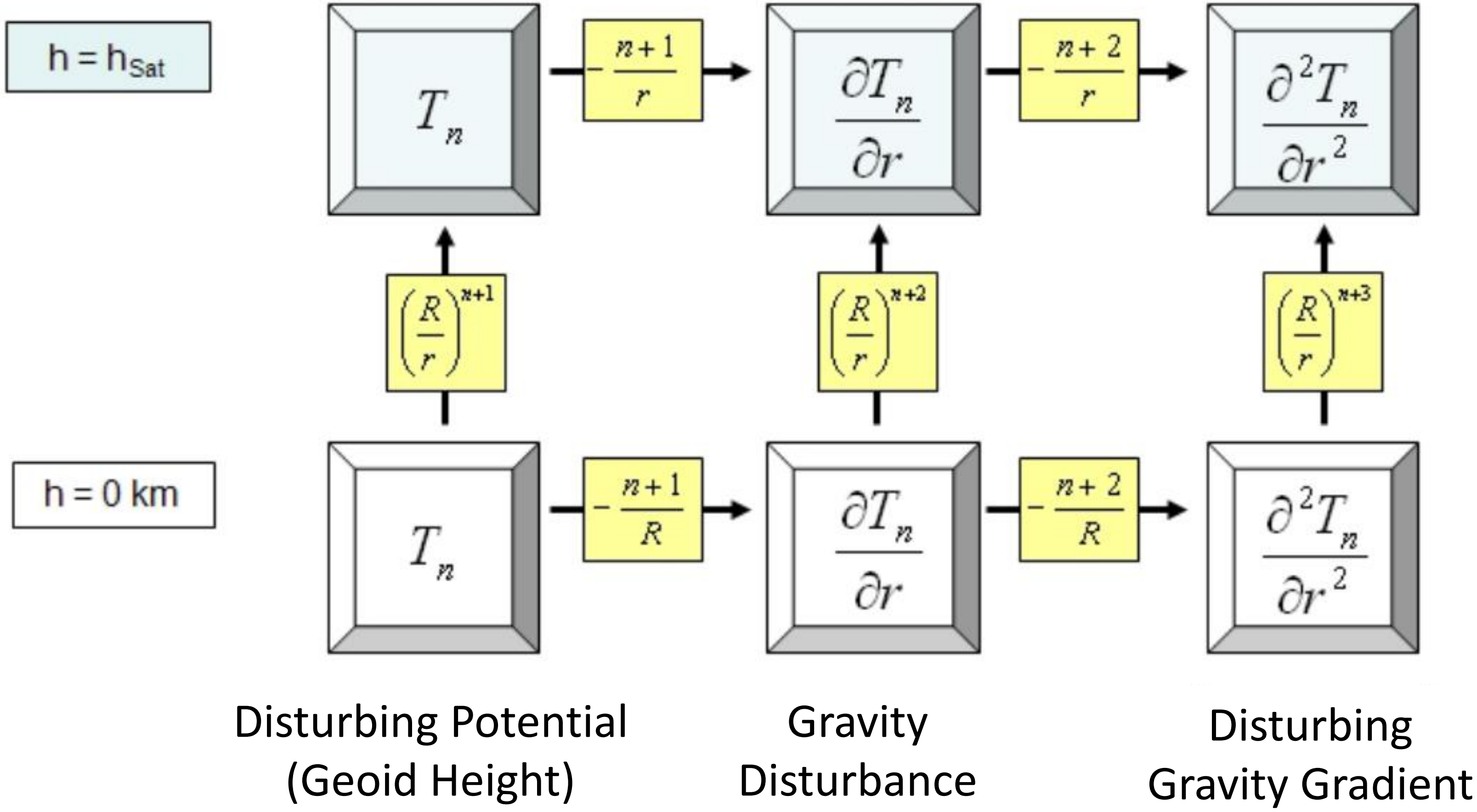
5. Signal Variances and Degree Variances



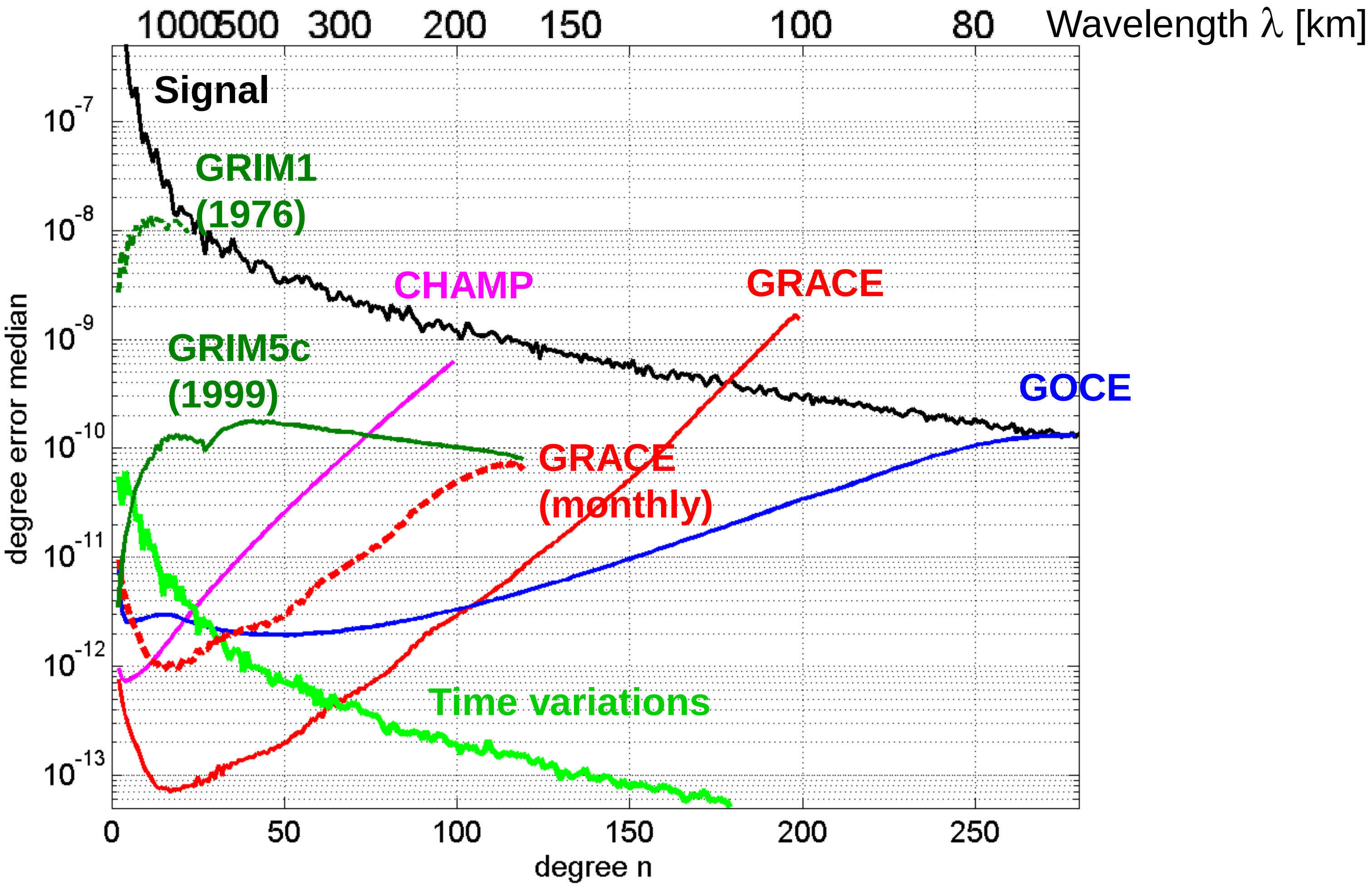
$$a_n = \sqrt{\frac{1}{2n+1}\sum_{m=0}^{n}\left(\bar{C}_{nm}^2 + \bar{S}_{nm}^2\right)}$$

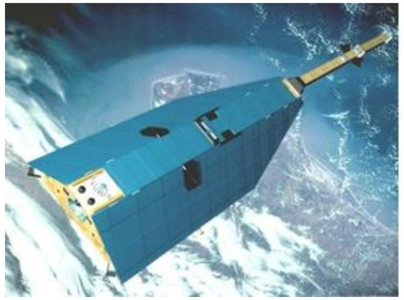
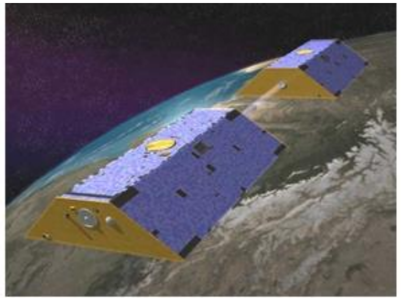
$$\sigma_n = \sqrt{\frac{1}{2n+1}\sum_{m=0}^{n}\left(\sigma_{\bar{C}_{nm}}^2 + \sigma_{\bar{S}_{nm}}^2\right)}$$

Meissl Scheme - Damping & Amplification



5. Gravity Signal and Noise




$$\lambda\ [km] = \frac{20000\ km}{n}$$

Why do we use spherical harmonics for the global representation of Earth‘s gravity field ?

S: Because they are a pain in the ass for many students

A: Because they are orthogonal also in discrete form

R: Because they are a special solution of Laplace equation

T: Because they are stationary and ergodic

What does the maximum degree of the SH expansion physically mean ?

E: There is a lack of base functions

O: It gives the number of zeros in North-South direction

I: It is closely related to the temporal behaviour of the field

A: It determines the maximum spatial resolution of the resulting field

Why is the height of the satellite relevant for the achievable performance ?

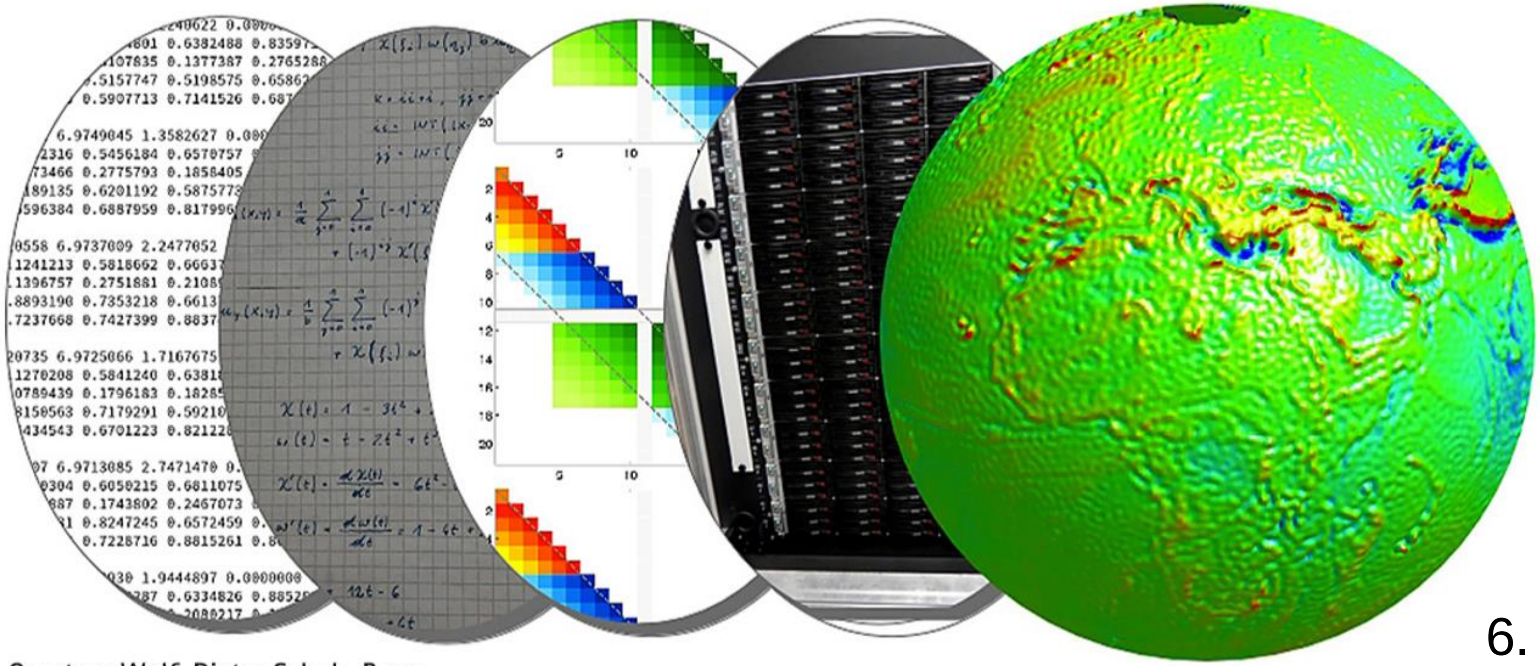
C: Because with increasing altitude the high-frequency signals are damped

E: Because in higher altitudes to orbit determination of the satellite is more difficult

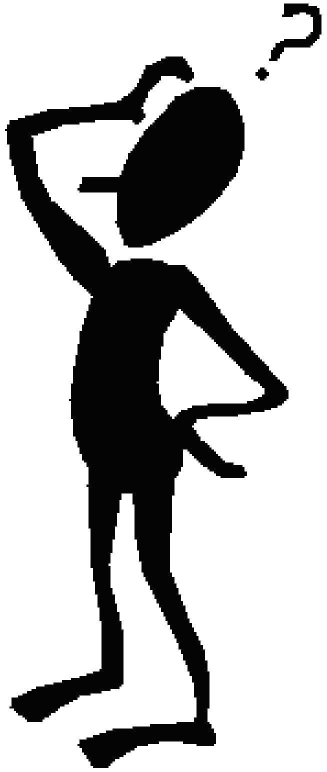
D: Because at lower altitudes the increased drag has negative impact on performance

R: Because at higher altitudes the gravitational attraction is close to zero

How can we determine a Global Gravity Model from Satellite Observations?

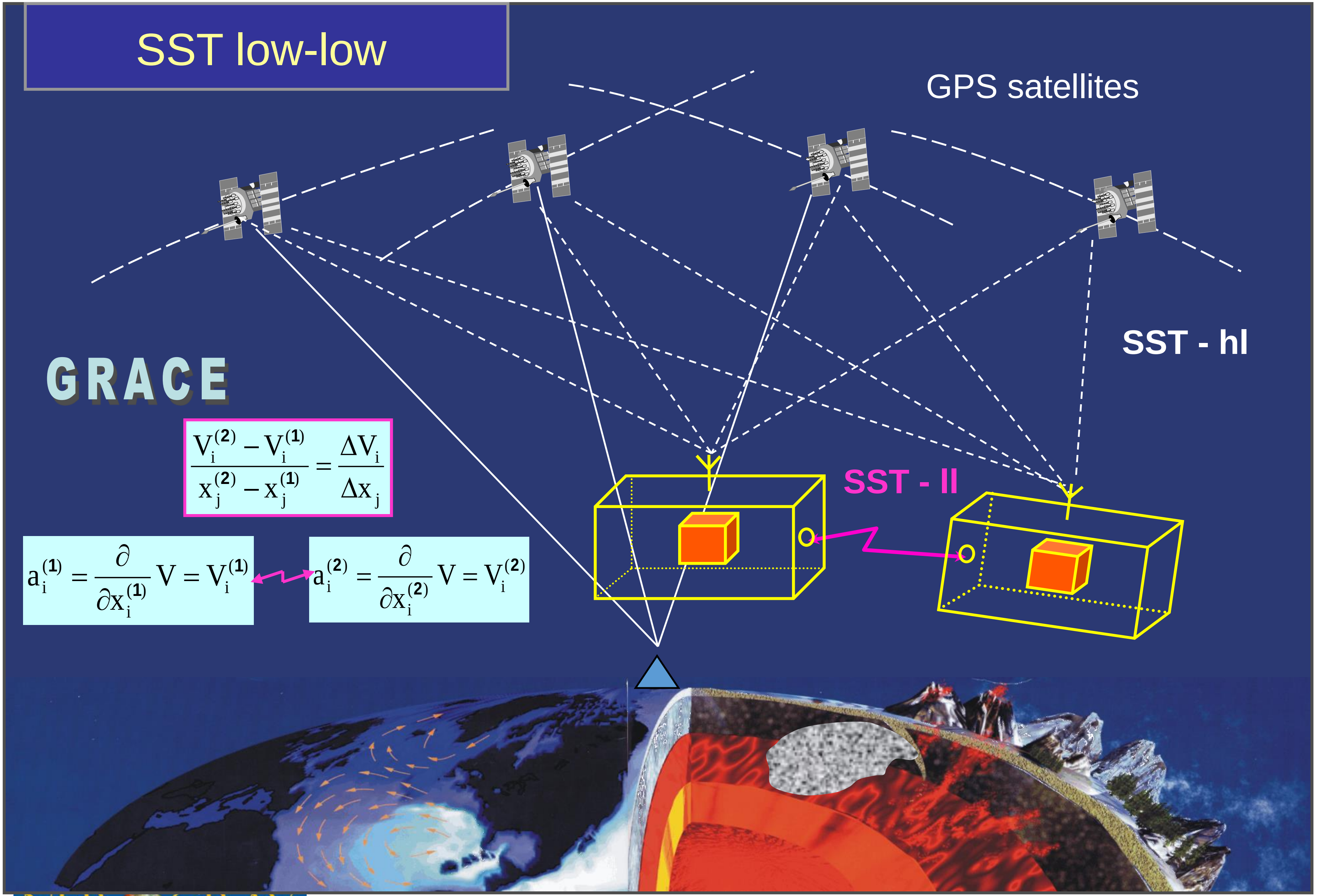


Courtesy Wolf-Dieter Schuh, Bonn



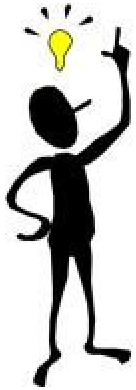
6. High-level Processing Overview
7. Specific Aspects
8. Alternatives to Spherical Harmonics

6. GRACE Measurement Principle



Key observables:

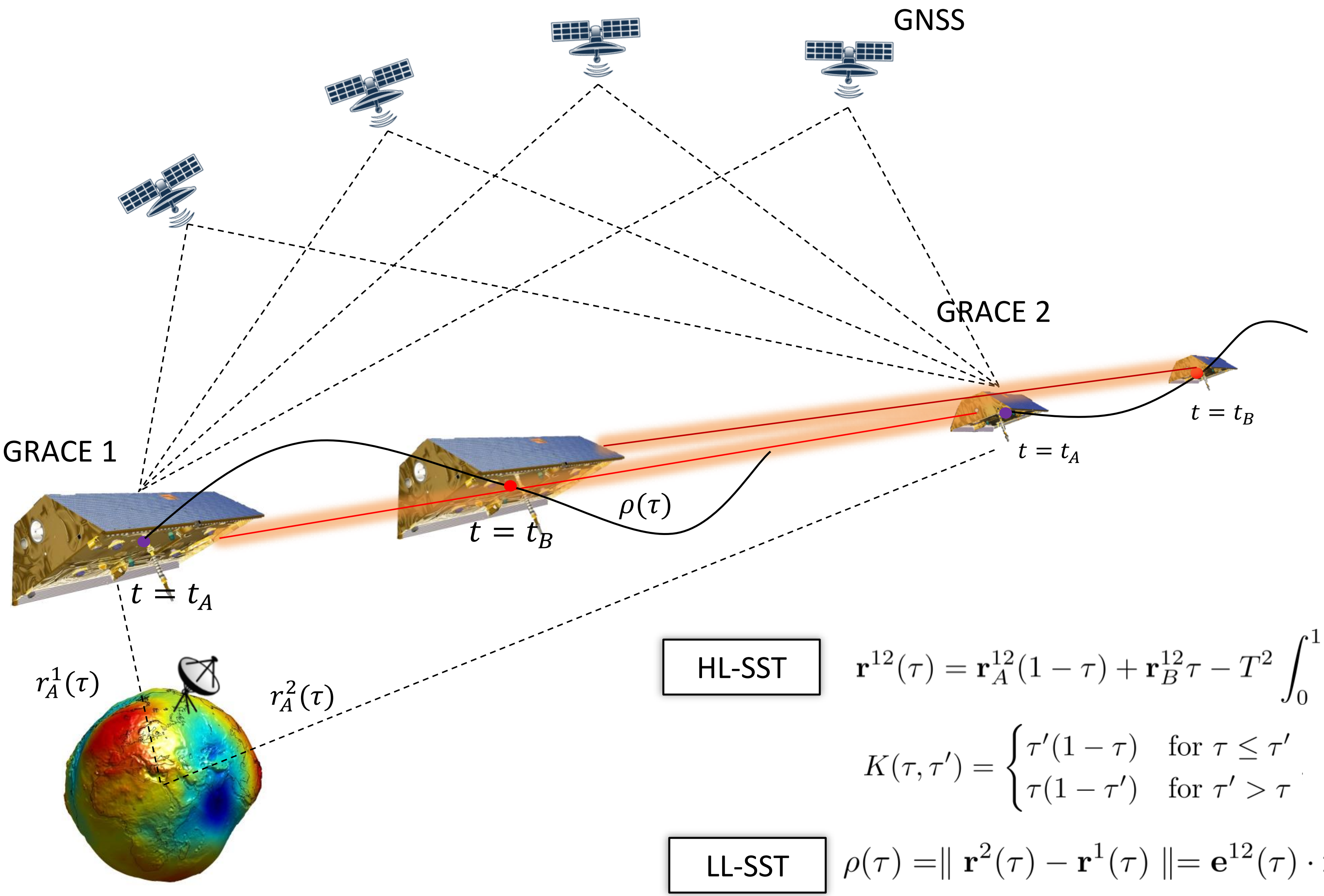
- Inter-satellite ranging
- GPS orbits



We want to derive a physical quantity (mass/gravity) from a geometrical quantity (inter-satellite distance [change]):

- not direct functional of gravity potential
- highly non-linear

6. GRACE Measurement Principle



HL-SST

$$\mathbf{r}^{12}(\tau) = \mathbf{r}_A^{12}(1-\tau) + \mathbf{r}_B^{12}\tau - T^2 \int_0^1 K(\tau,\tau')\mathbf{f}^{12}(\mathbf{r}(\tau'))d\tau',$$

$$K(\tau,\tau') = \begin{cases} \tau'(1-\tau) & \text{for } \tau \le \tau' \\ \tau(1-\tau') & \text{for } \tau' > \tau \end{cases}$$

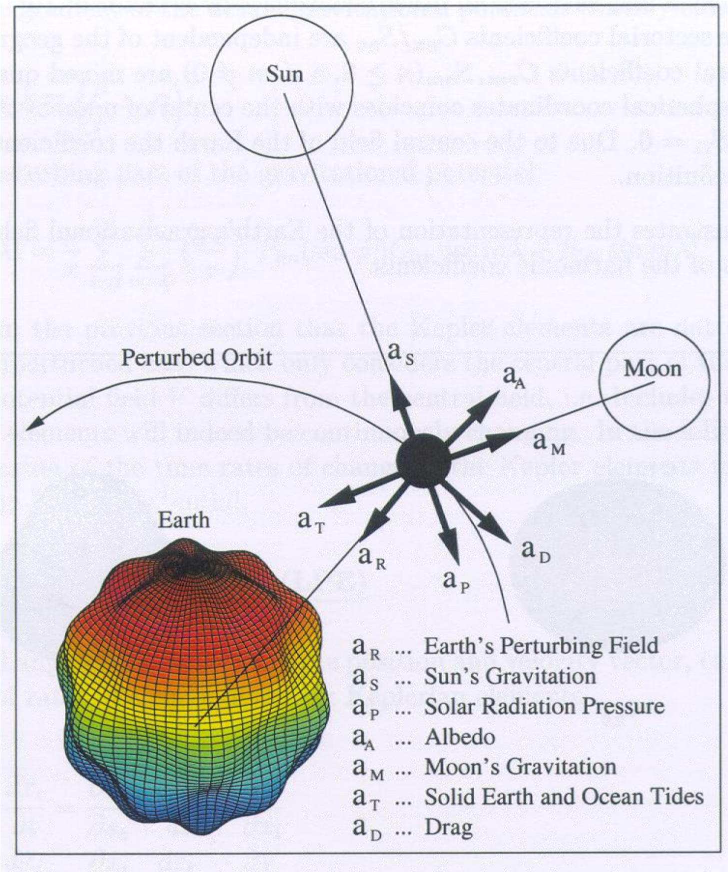
LL-SST

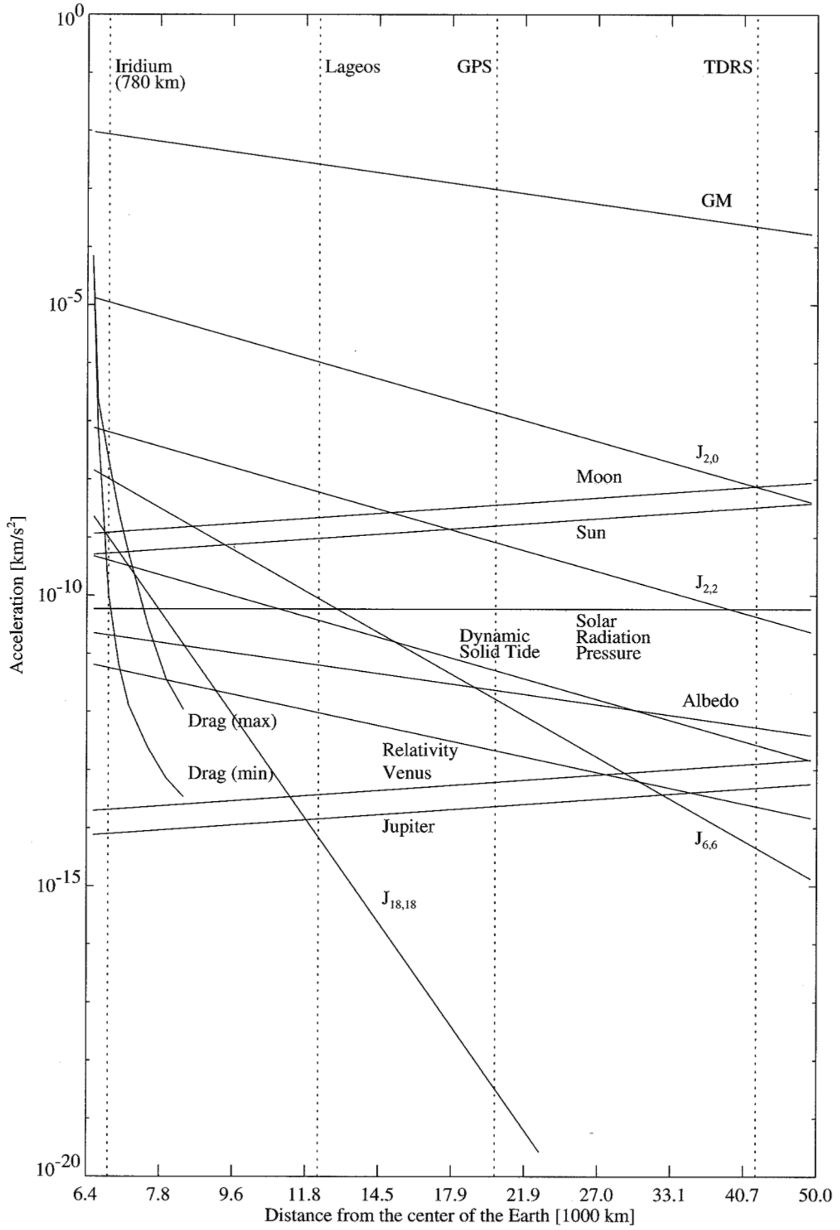
$$\rho(\tau) = \| \mathbf{r}^2(\tau) - \mathbf{r}^1(\tau) \| = \mathbf{e}^{12}(\tau) \cdot \mathbf{r}^{12}(\tau)$$

6. Orbit Perturbations

Equation of motion

$$\ddot{\underline{r}} = \underline{a}_c + \underline{a}_{nc} = \nabla V + \underline{a}_{nc} = \nabla\left(\frac{GM}{r}\right) + \underline{d}$$

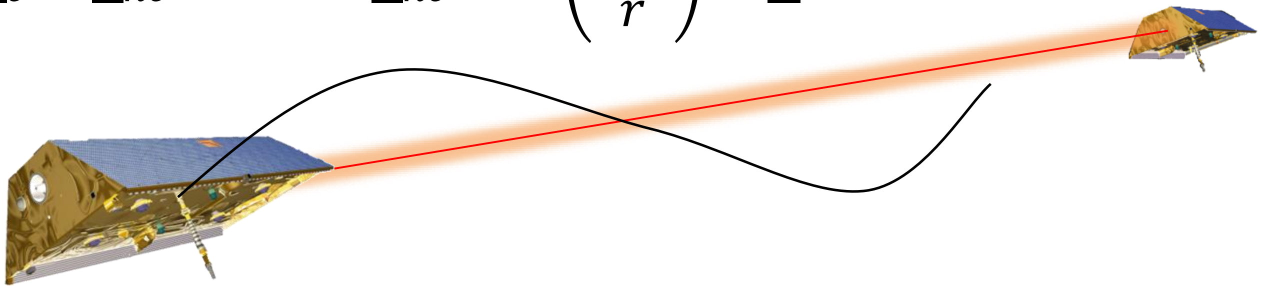




6. GRACE Observation Equation

Equation of motion

$$\ddot{\underline{r}} = \underline{a}_c + \underline{a}_{nc} = \nabla V + \underline{a}_{nc} = \nabla\left(\frac{GM}{r}\right) + \underline{d}$$



Observations:

- (biased) range ρ
- range rate $\dot{\rho}$
- range acceleration $\ddot{\rho}$

Numerical orbit integration → position+velocity → range/range rate

Observation equation

$$y(t) = f\big(t, \overbrace{\underline{r}, \dot{\underline{r}}}^{\text{Orbit}}, \underline{x}, \underline{d}, \underbrace{cal, emp}, \dots\big) = f_0\big(t, \underbrace{\underline{r}_0, \dot{\underline{r}}_0, \underline{x}_0}, \dots\big) + \delta f$$

Orbit

Force models (ACC meas.) → $\underline{d}$

$\underline{x}$ ↑ Gravity field parameters

cal, emp: Calibration/empir. parameters

$\underline{r}_0, \dot{\underline{r}}_0, \underline{x}_0$: A-priori models

δf ↑ Obs. Residuals → input to adjustment

$$\underline{x} = \{\overline{C}_{nm}, \overline{S}_{nm}\}$$

6. GRACE Observation Equation

Observation equation

$$y(t) = f\big(t, \underline{r}, \underline{\dot{r}}, \underline{x}, \underline{d}, cal, emp, \dots\big) = f_0\big(t, \underline{r}_0, \underline{\dot{r}}_0, \underline{x}_0, \dots\big) + \delta f$$

Gravity field parameters ($\underline{x}$)

Calibration/empirical parameters (cal, emp)

Orbit ($\underline{r}, \underline{\dot{r}}$)

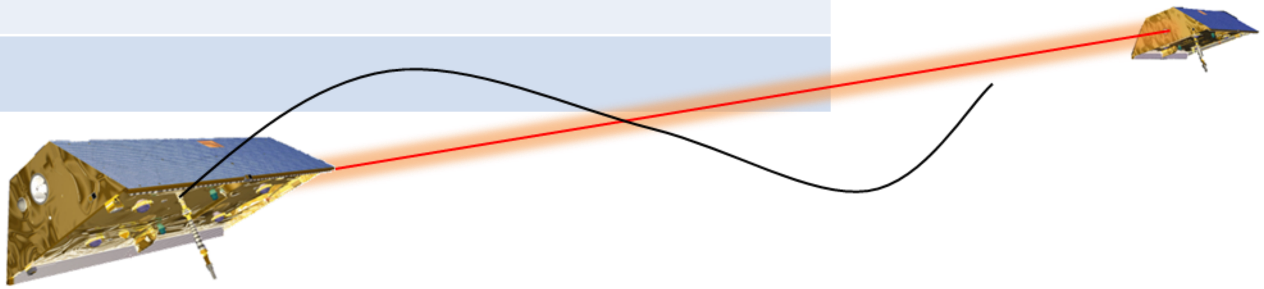
Force models (ACC meas.) ($\underline{d}$)

A-priori models ($\underline{r}_0, \underline{\dot{r}}_0, \underline{x}_0$)

Obs. Residuals (δf)
→ input to adjustment

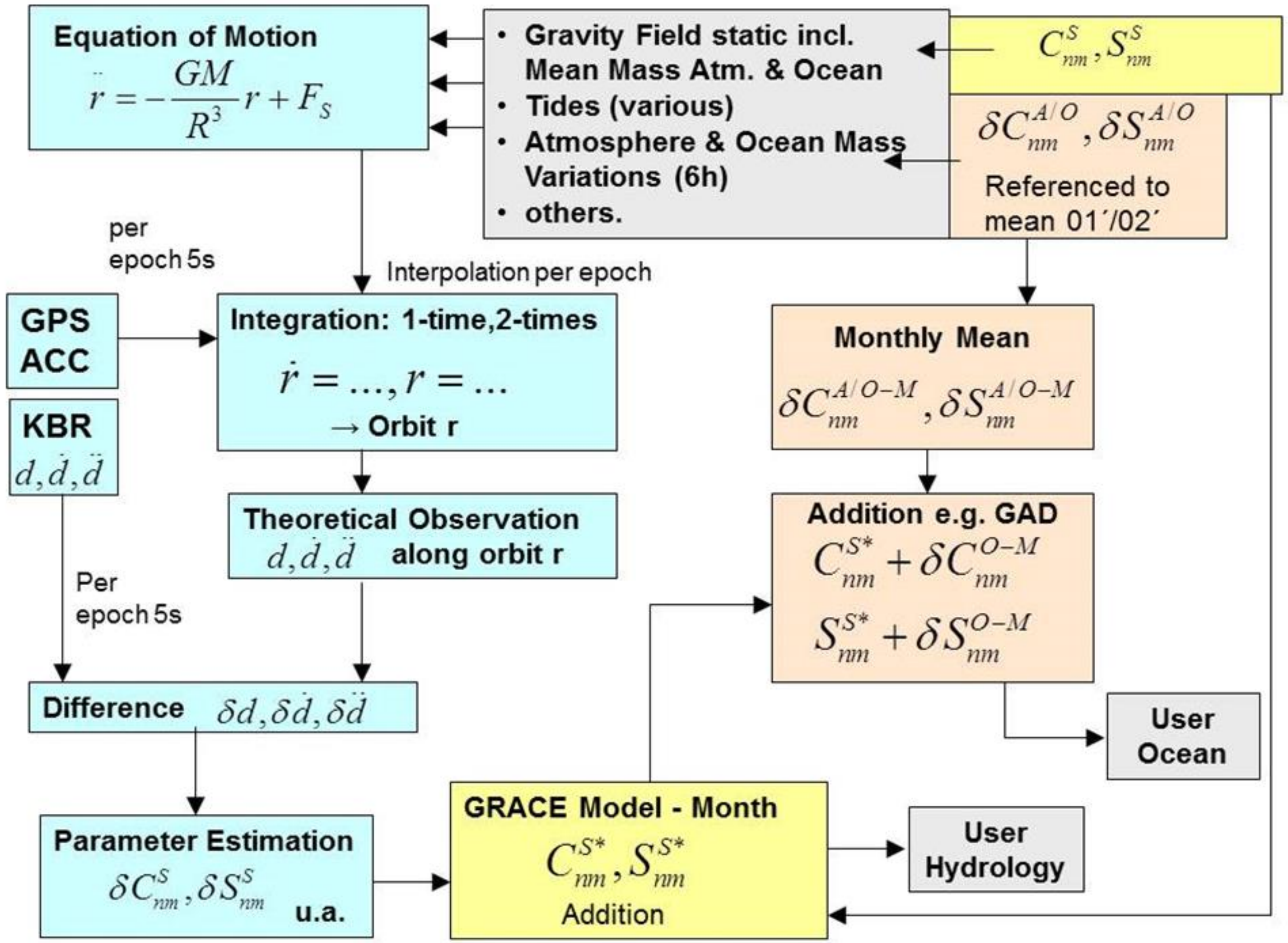
6. Methods of Earth's Gravity Field Recovery from GRACE Observations

Method	Observations	Reference
Variational equations	$\rho, \dot{\rho}$	Tapley et al. (2004), GRL
Celestrial mechanics approach	$\rho, \dot{\rho}$	Beutler et al. (2010), J. Geod.
Short-arc approach	$\rho, \dot{\rho}$	Mayer-Gürr et al (2006), Ph.D. Thesis
Energy balance approach	$\dot{\rho}$	Han et al. (2006), J. Geophys. Res.
Acceleration approach	$\ddot{\rho}$	Liu (2008), Ph.D. Thesis
Line of Sight Gradiometry	$\ddot{\rho}/\rho$	Keller & Sharifi (2005), J. Geod.
…		

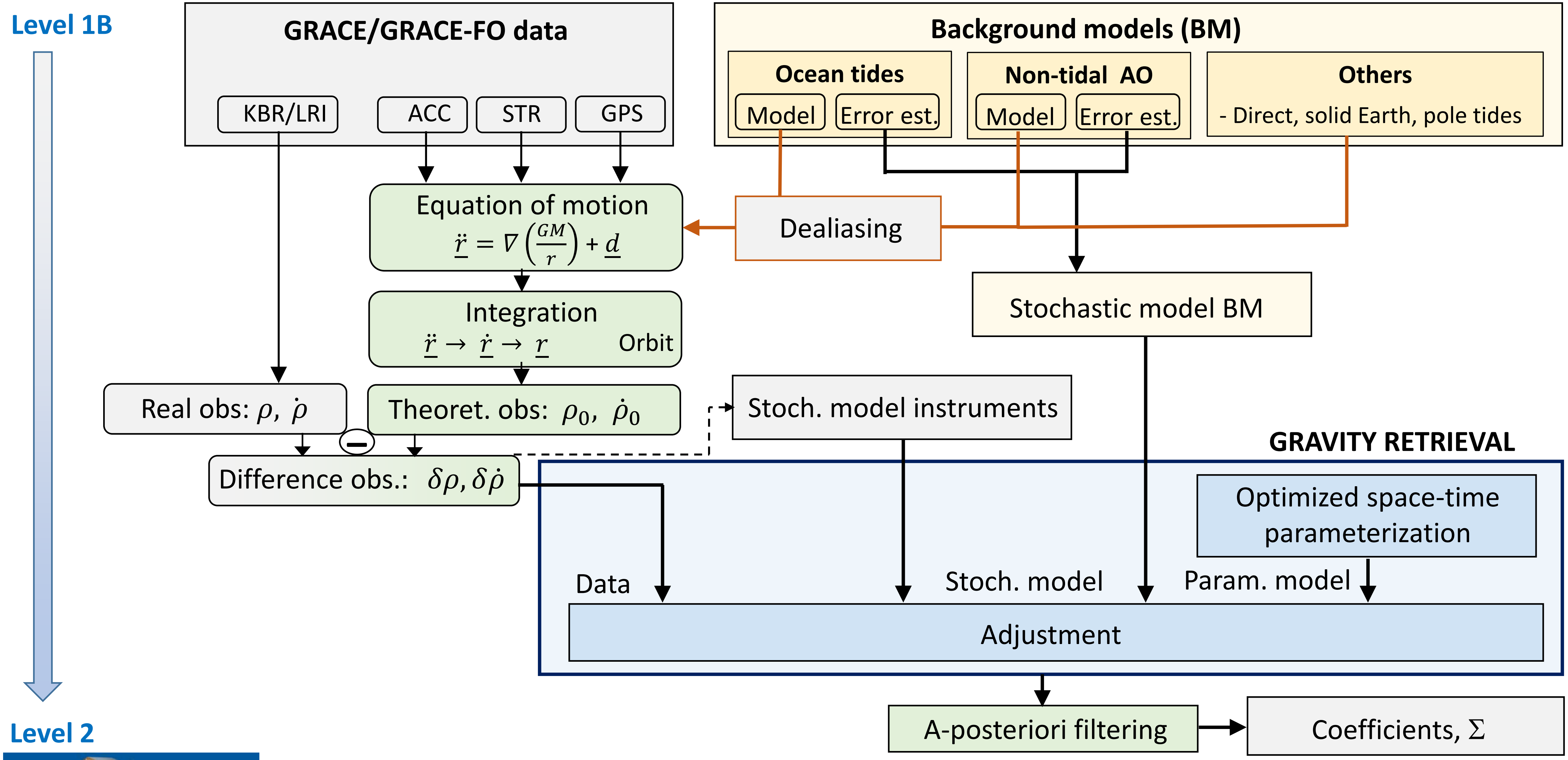


6. Gravity Field Processing – Overview (Status 2007)

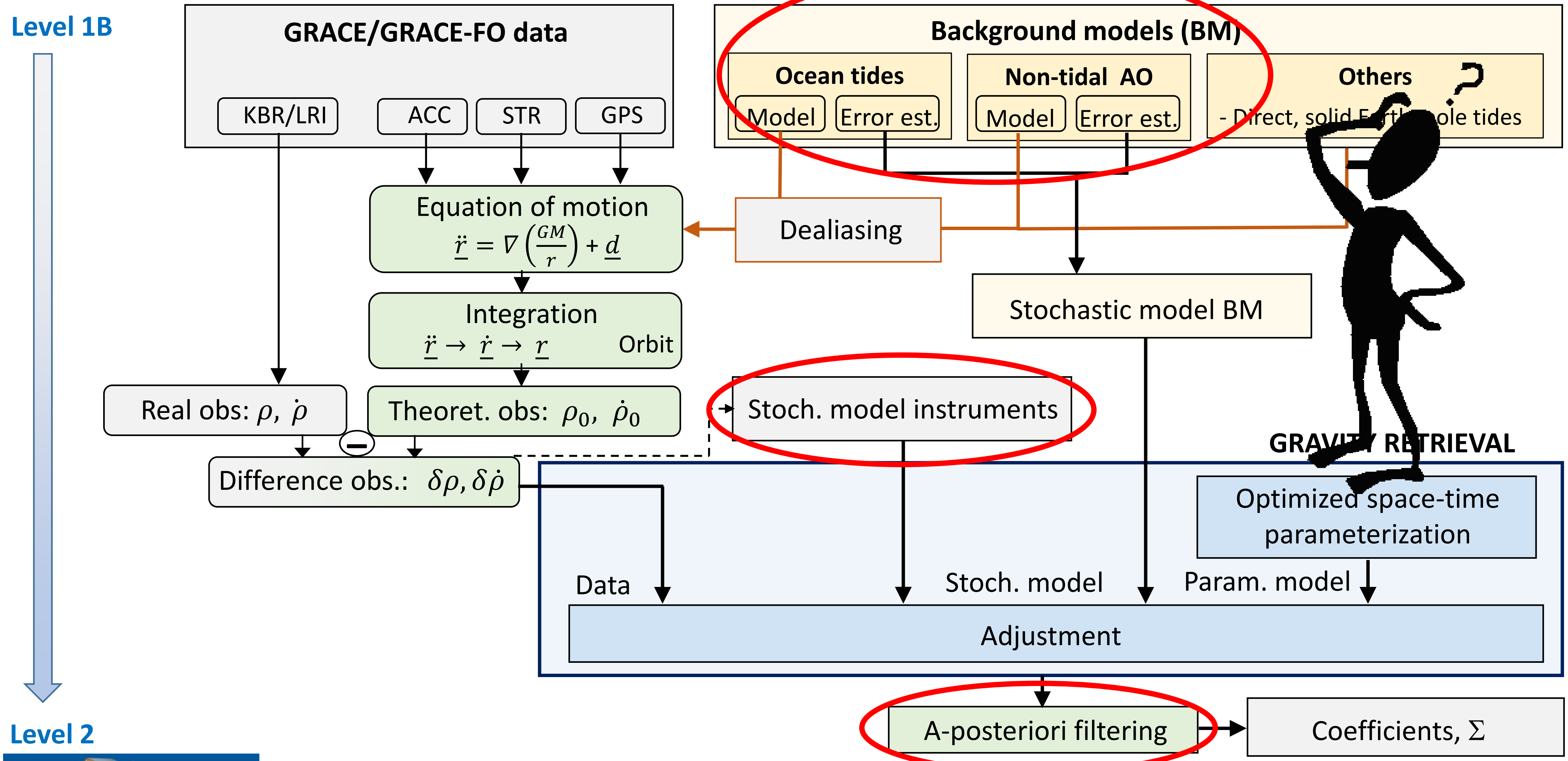
prepared by Thomas Gruber and Frank Flechtner for 2007 Workshop of DFG Priority Programme SPP1257 „Mass Transport and Mass Distribution in the System Earth“



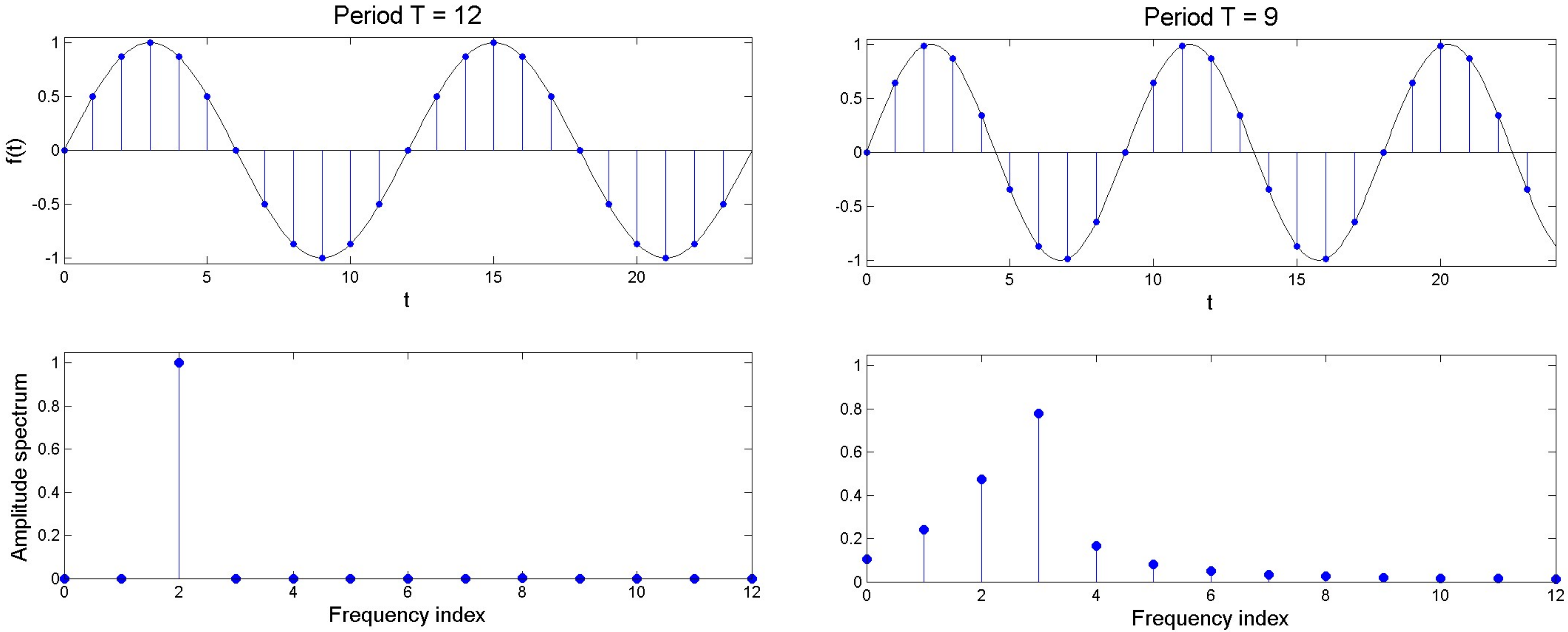
6. Gravity Field Processing – Overview



7. Specific Aspects

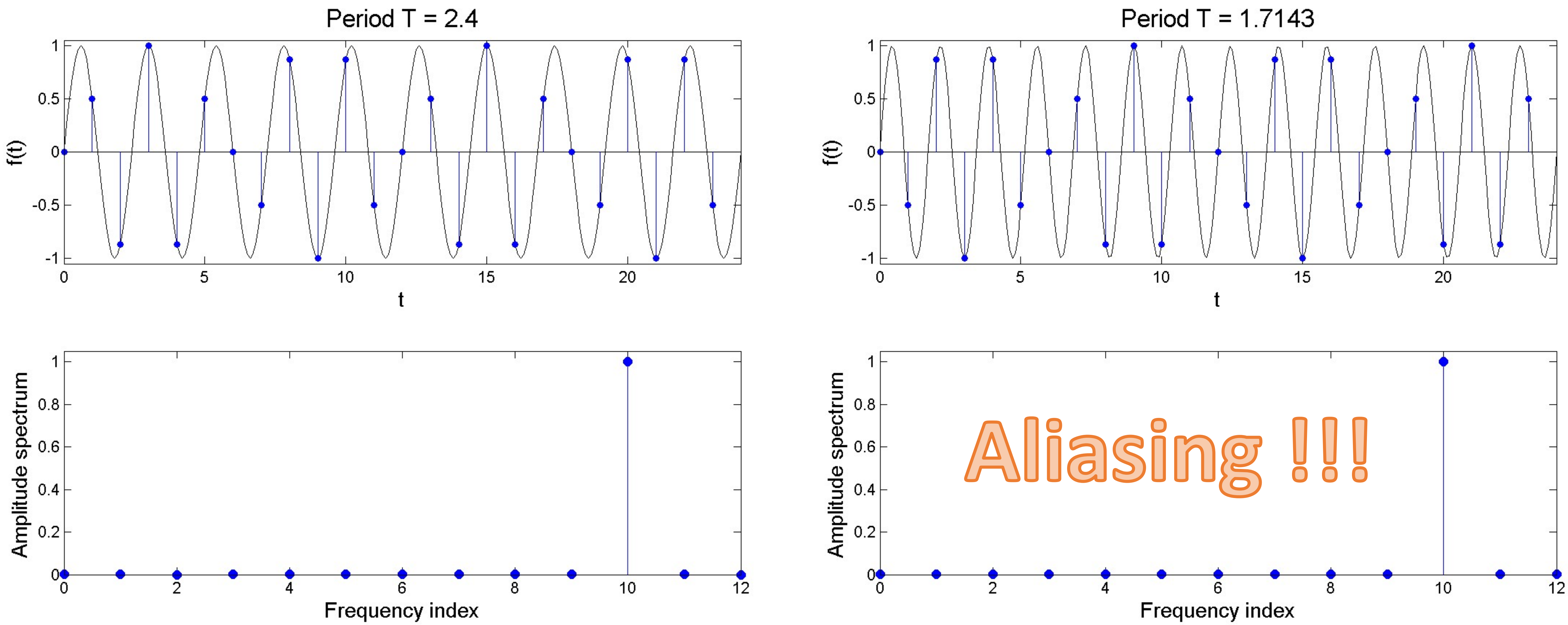


7. Aliasing: Fourier Analysis of sine Function



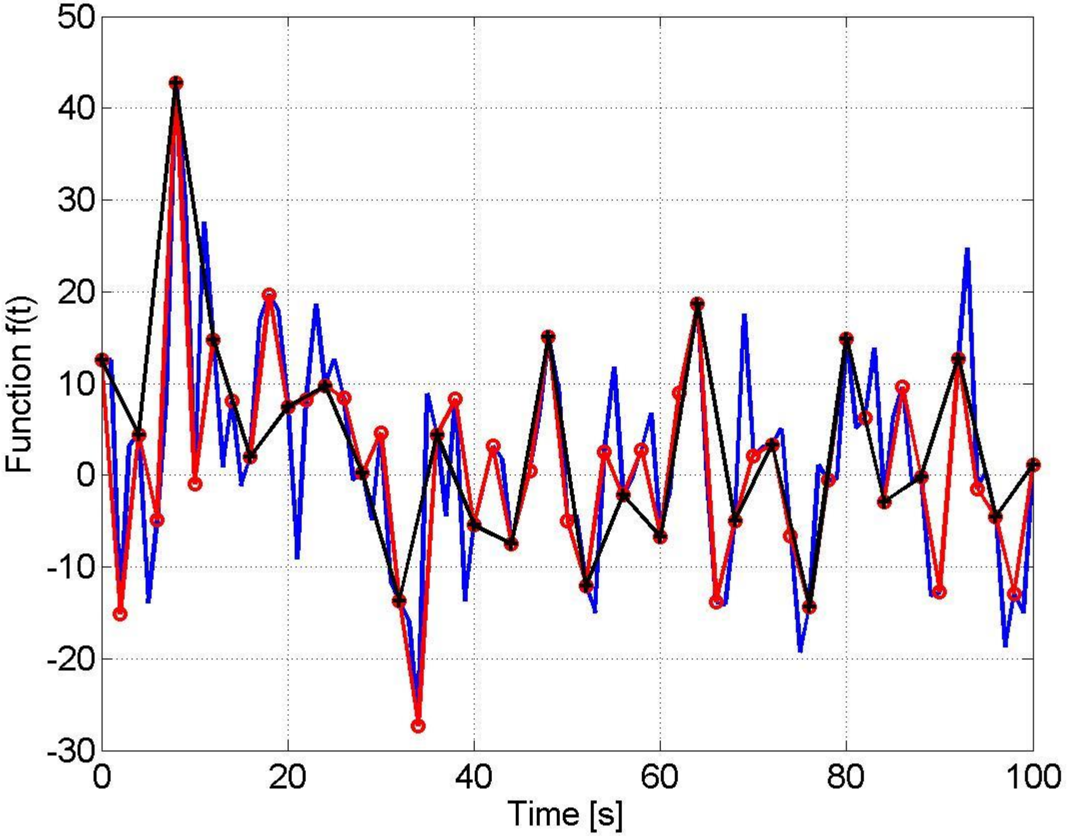
- Just one spectral line (freq. index 2) non-equal zero
- Several spectral lines, because 24/9 is not integer

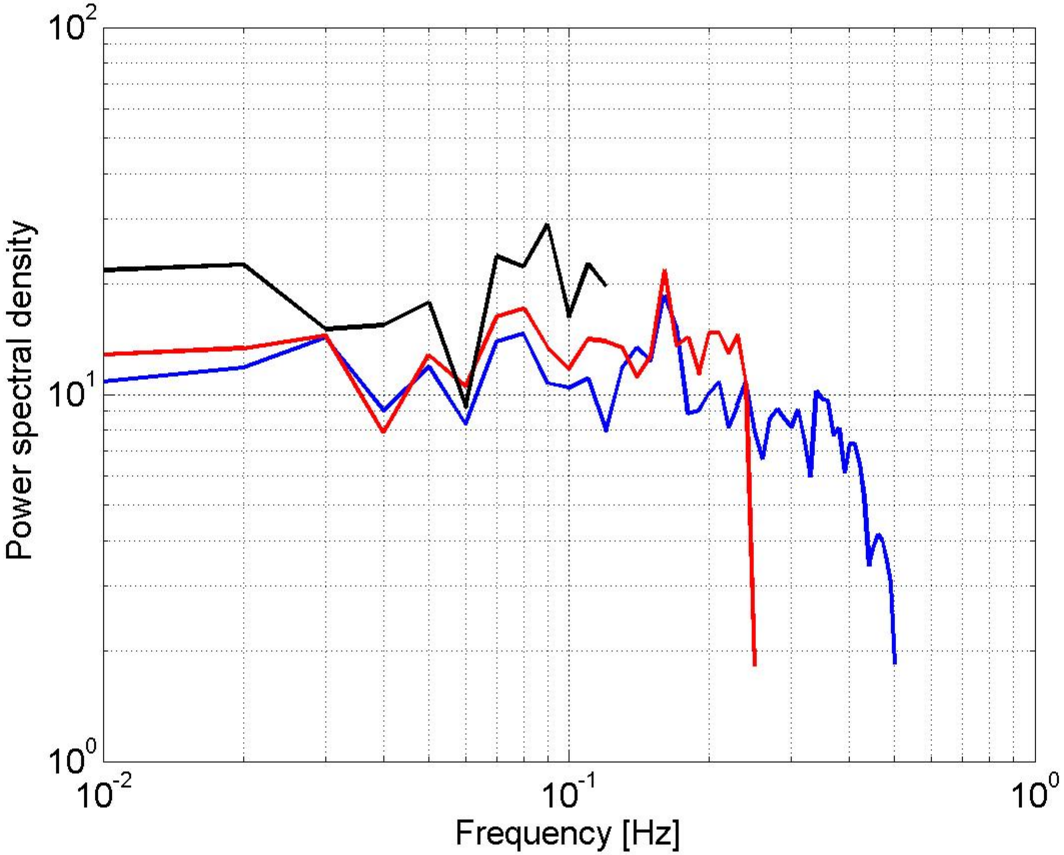
7. Aliasing: Fourier Analysis of sine Function



- Just one spectral line (freq. index 10) non-equal zero
- Same result as for period 2.4
- True signal (14 oscillations) cannot be recovered

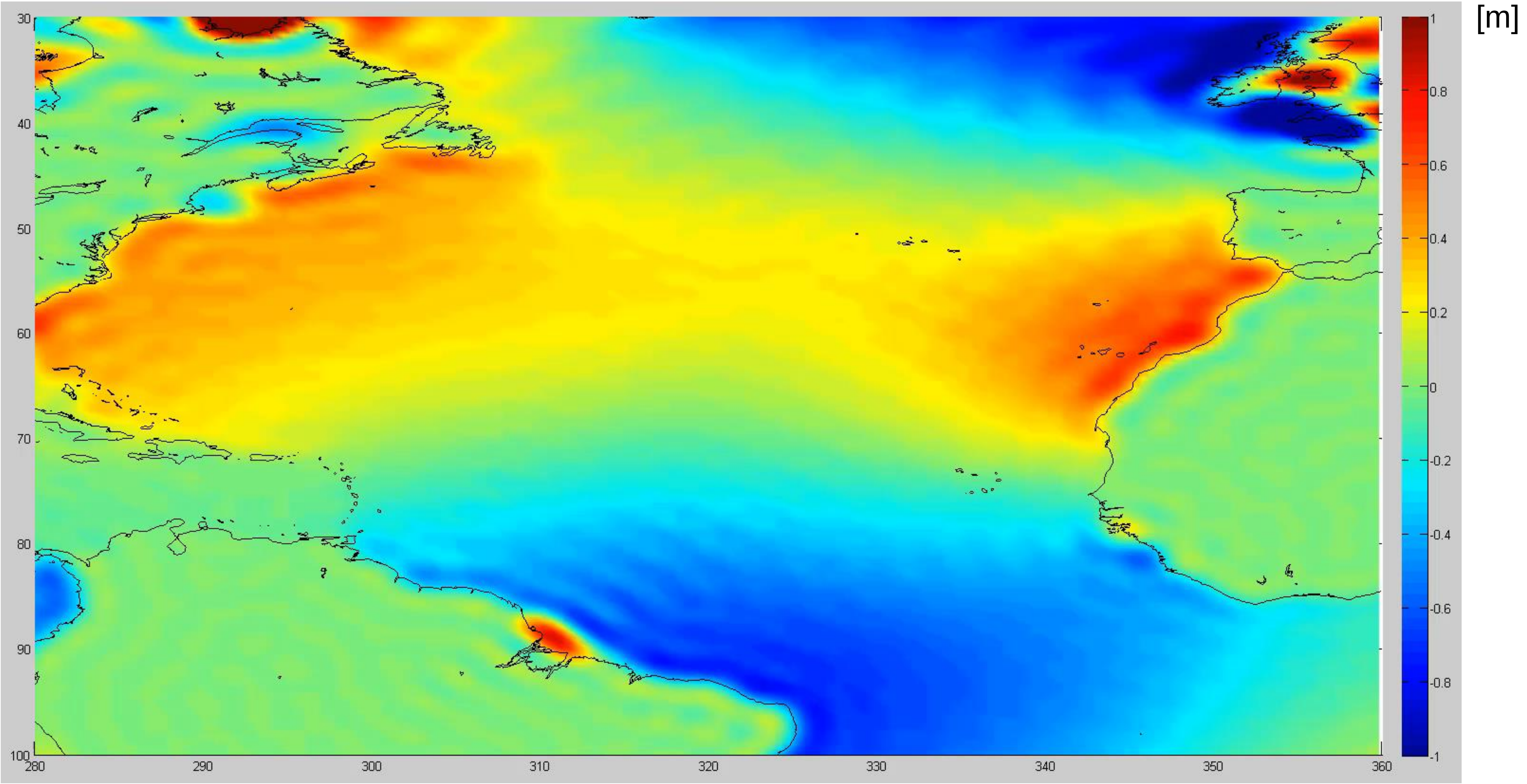
7. Aliasing: Example





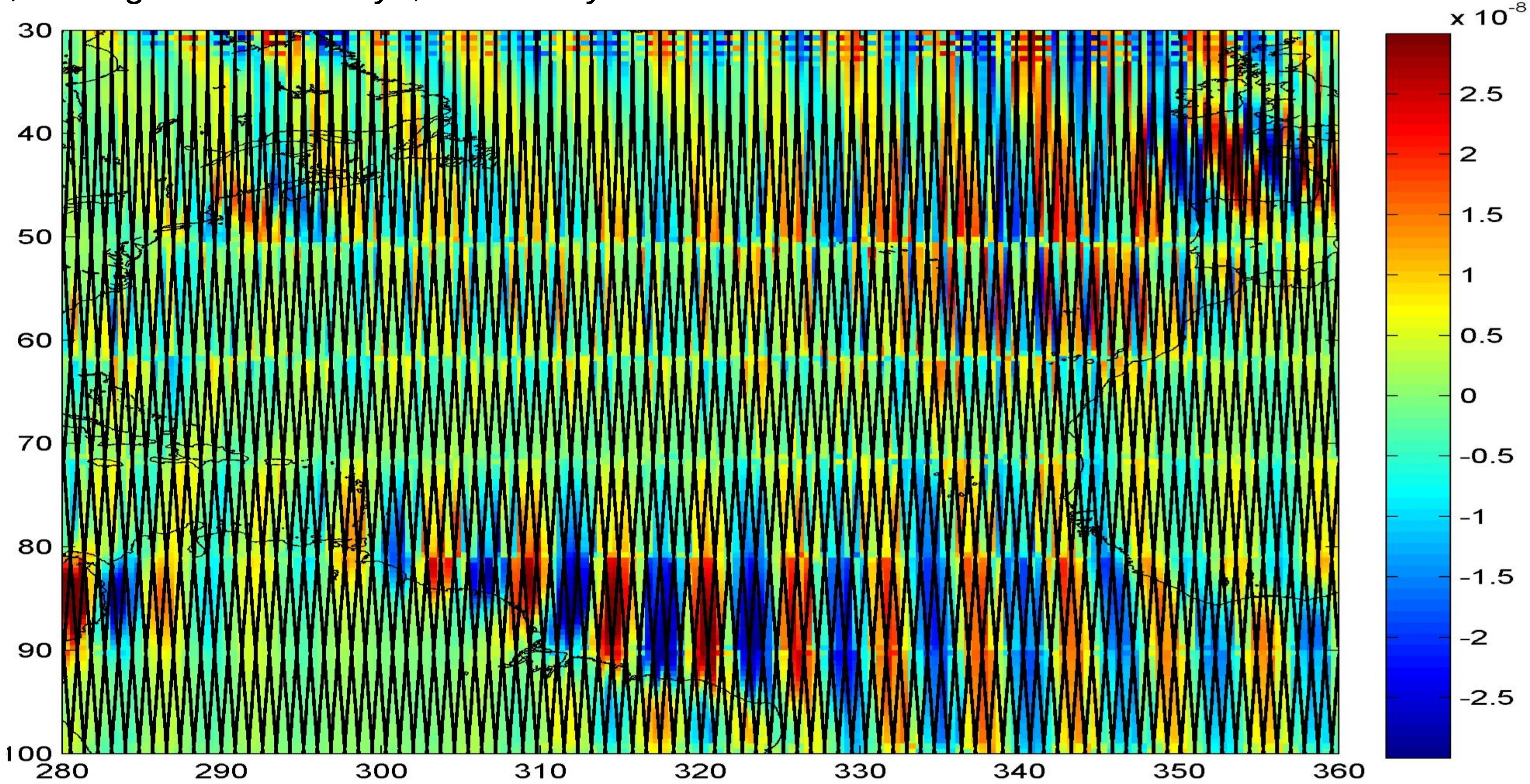
7. Aliasing: M2 Tide

M2 tidal heights, superimposed by satellite ground tracks



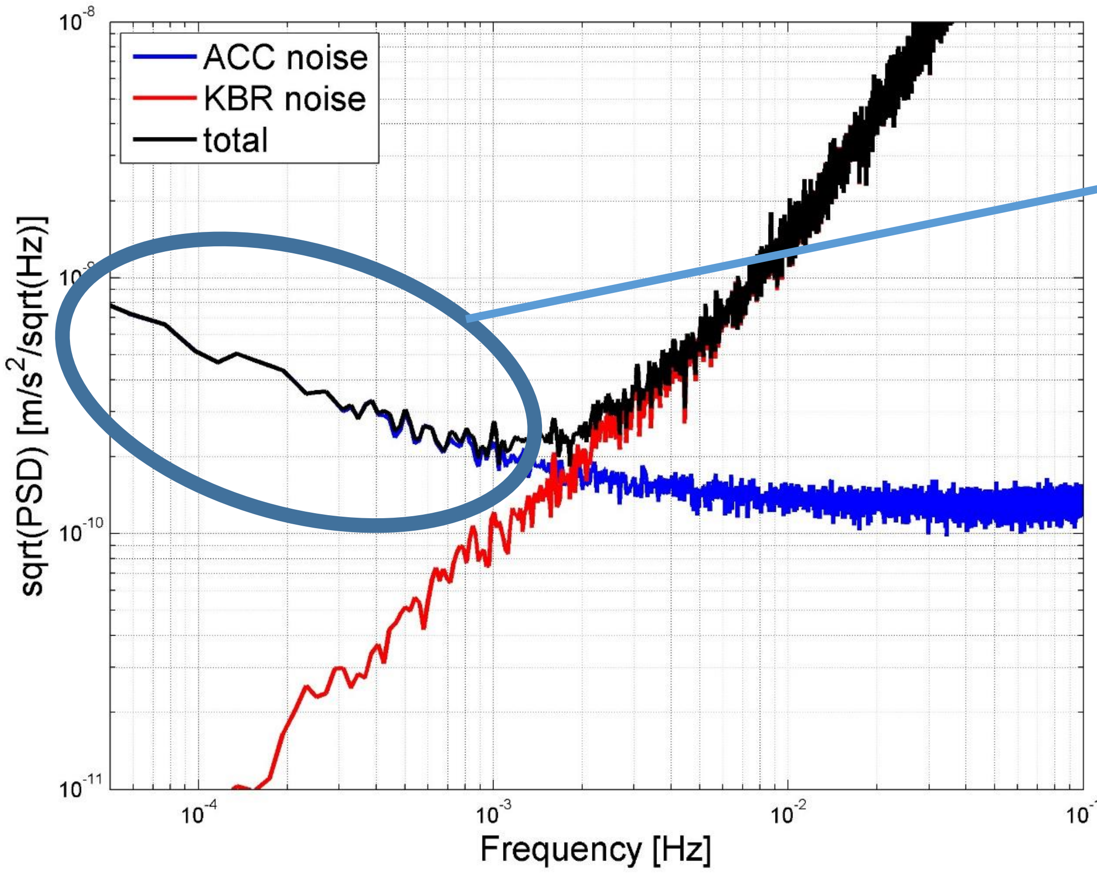
7. Aliasing: M2 Tide

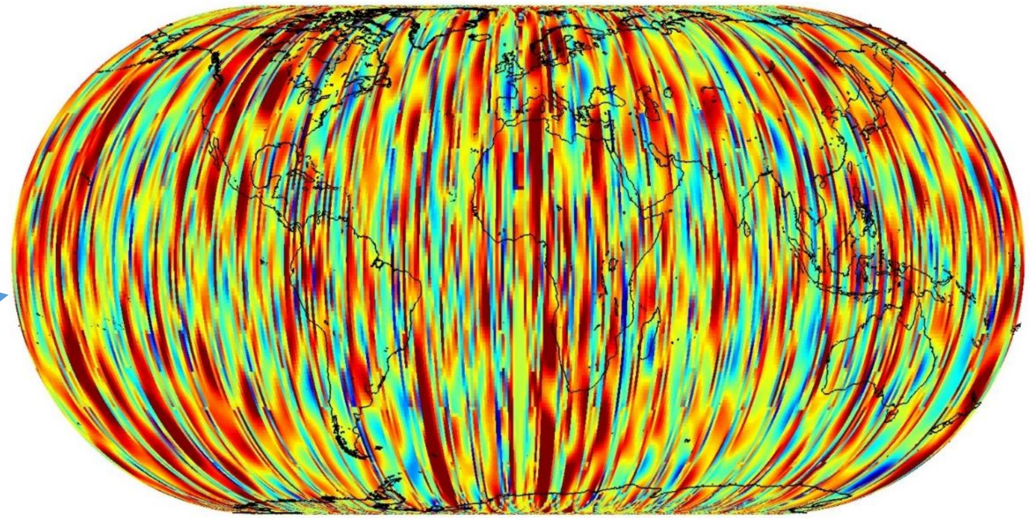
M2 signals, averages over 27 days, sensed by a satellite



7. Stochastic Modelling

Different performance of instruments as function of frequency





$$\Delta\hat{x} = (A^T\Sigma(y)^{-1}A)^{-1}\, A^T\, \Sigma(y)^{-1}\, y$$

Introduction of colored noise behaviour as stochastic model (weighting matrix)

7. Striping & Need for a-posteriori Filtering

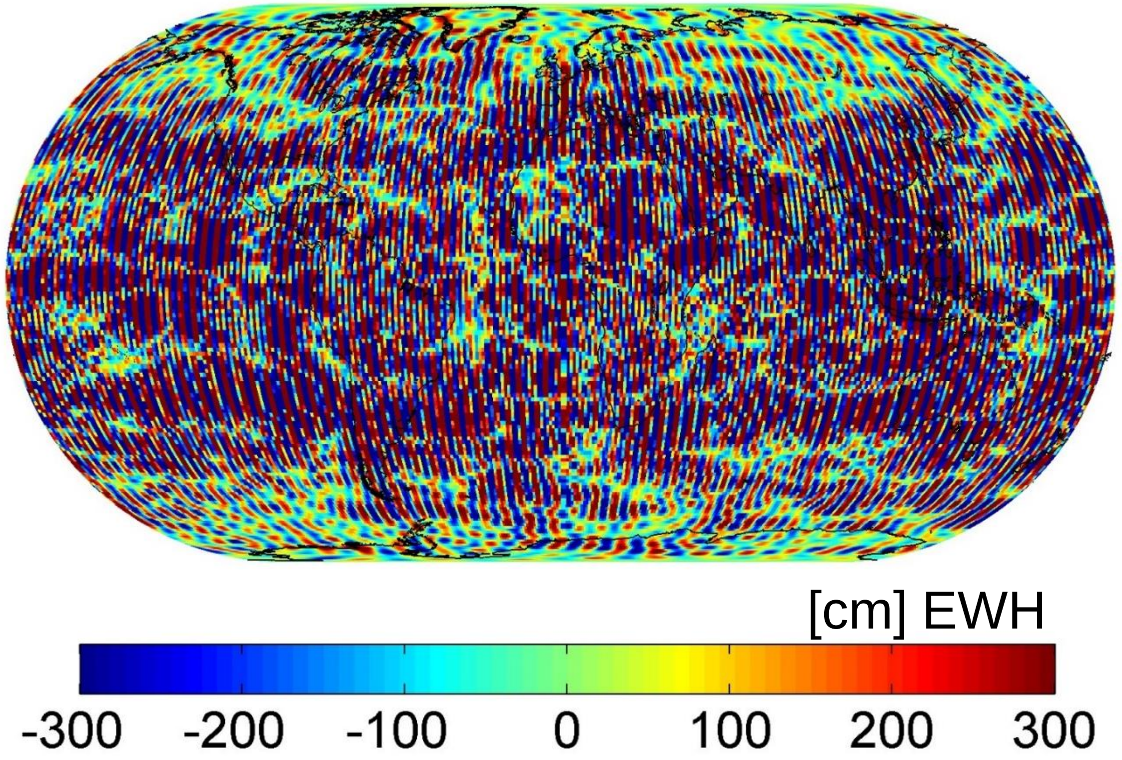
Striping results from

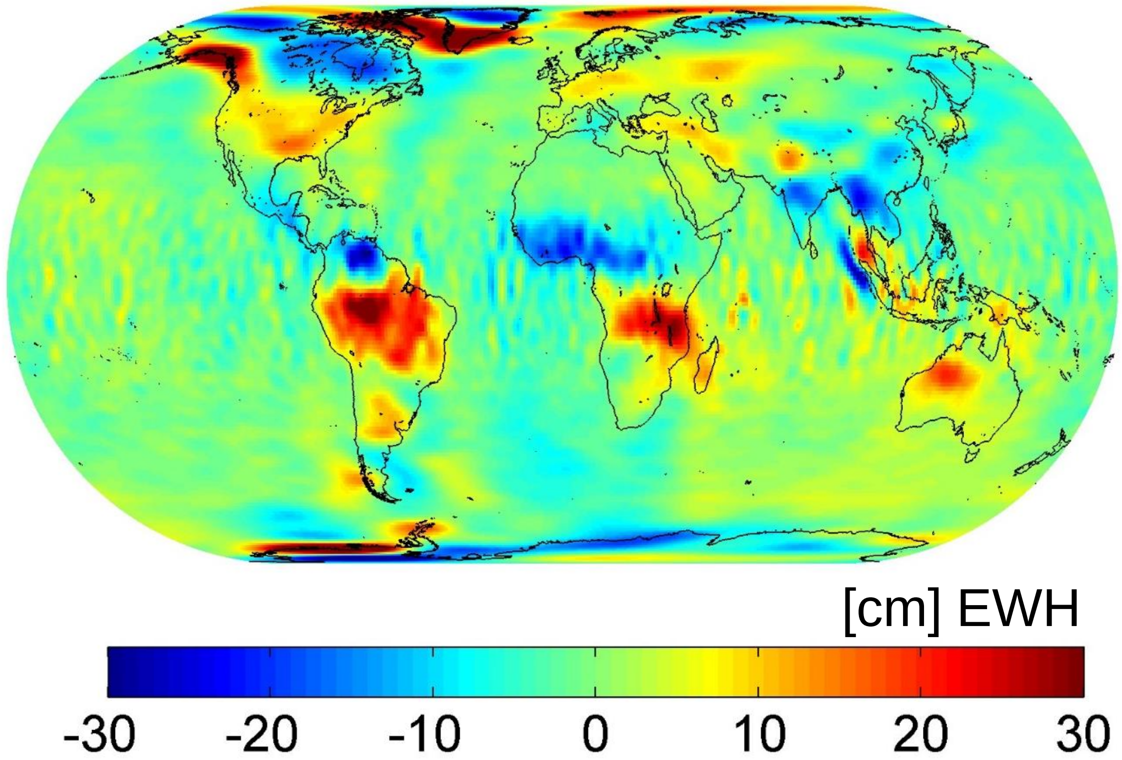
- anisotropic error behavior of along-track inter-satellite ranging
- colored noise behaviour of instruments
- temporal aliasing

FILTER

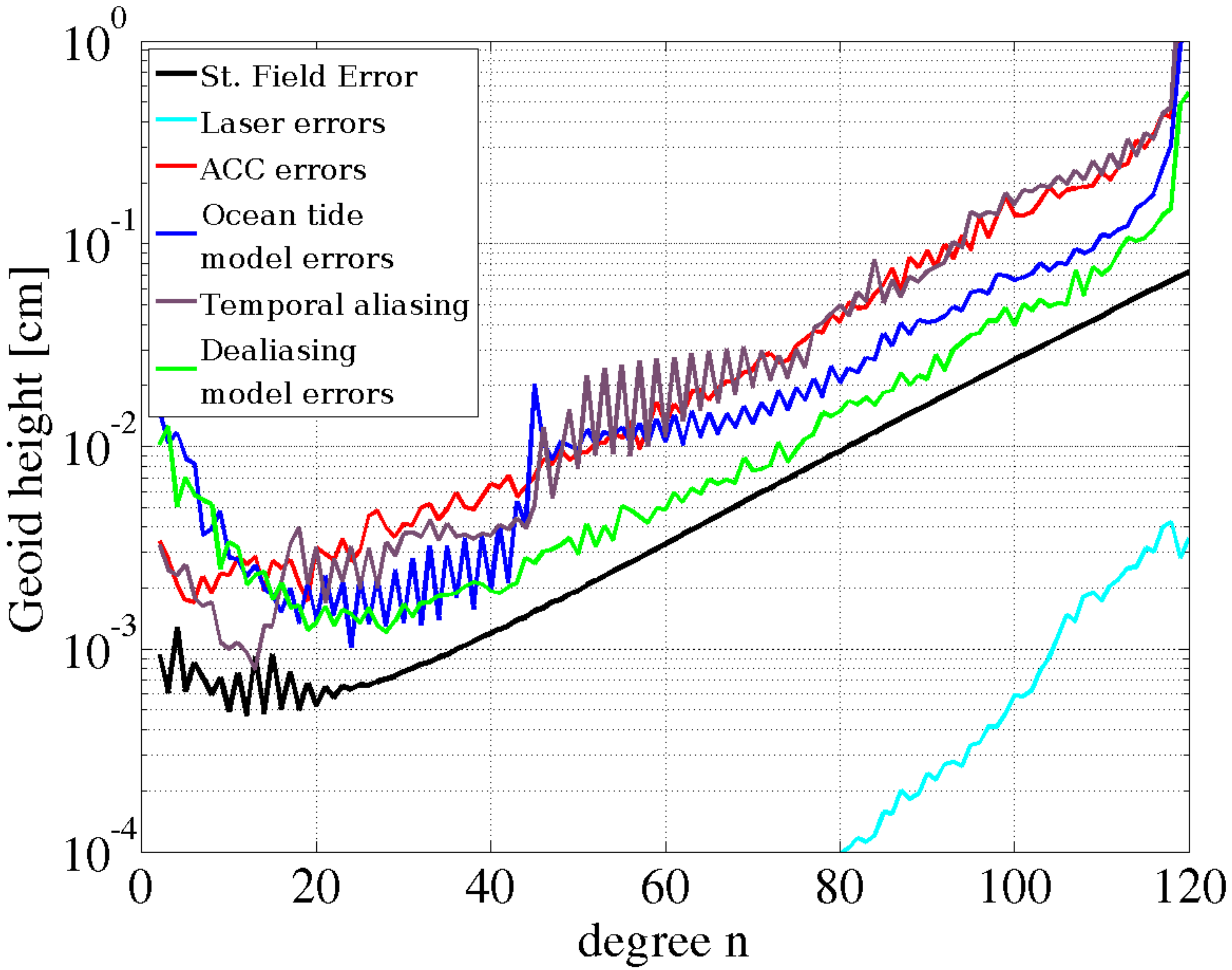
- Reduction of noise
- Reduction of signal (!)

→ „optimum“ filters





7. Contributors to Error Budget



Flechtner et al. (2017)

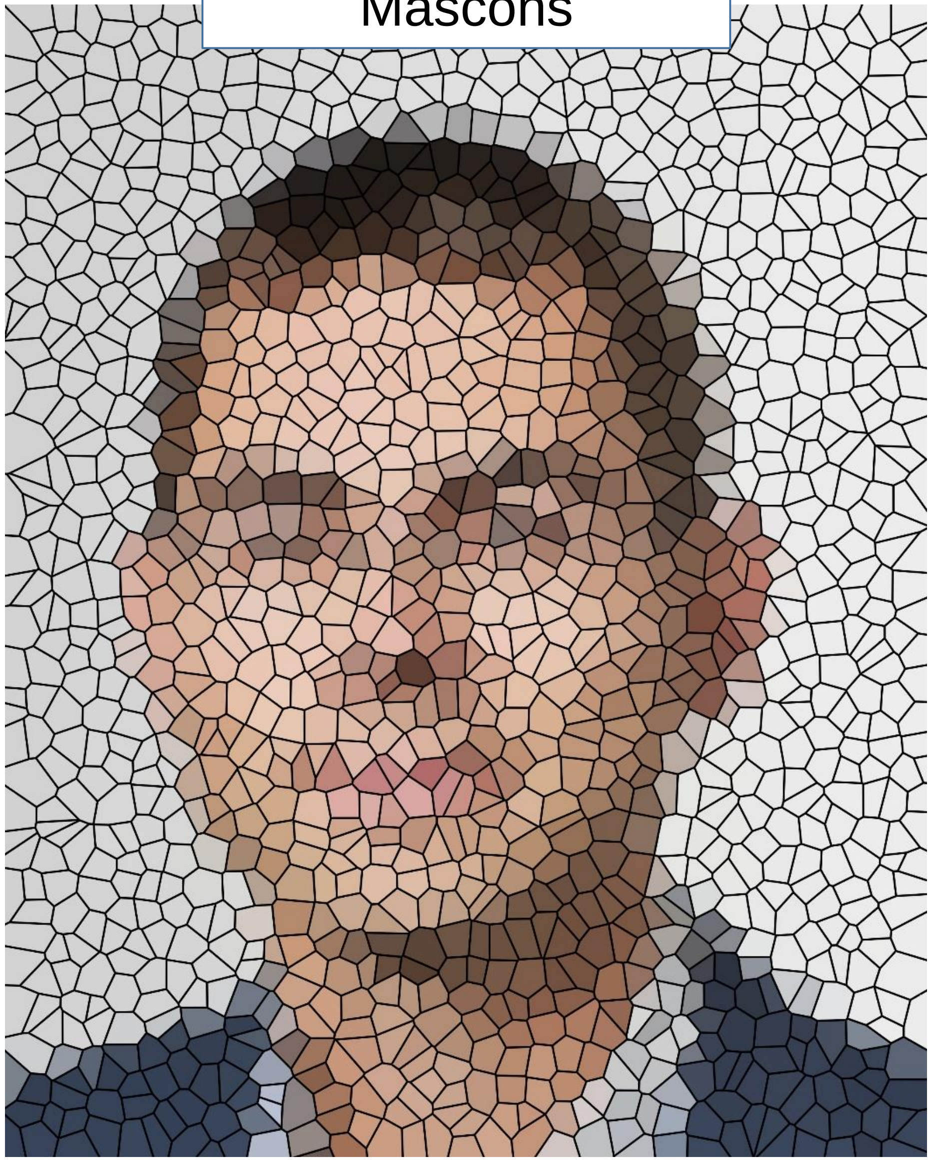
- Tidal aliasing of high-frequency signals
  - Ocean tides
  - AO signals
- Instruments
  - Accelerometer

8. Alternative Representations

Spherical harmonics

versus

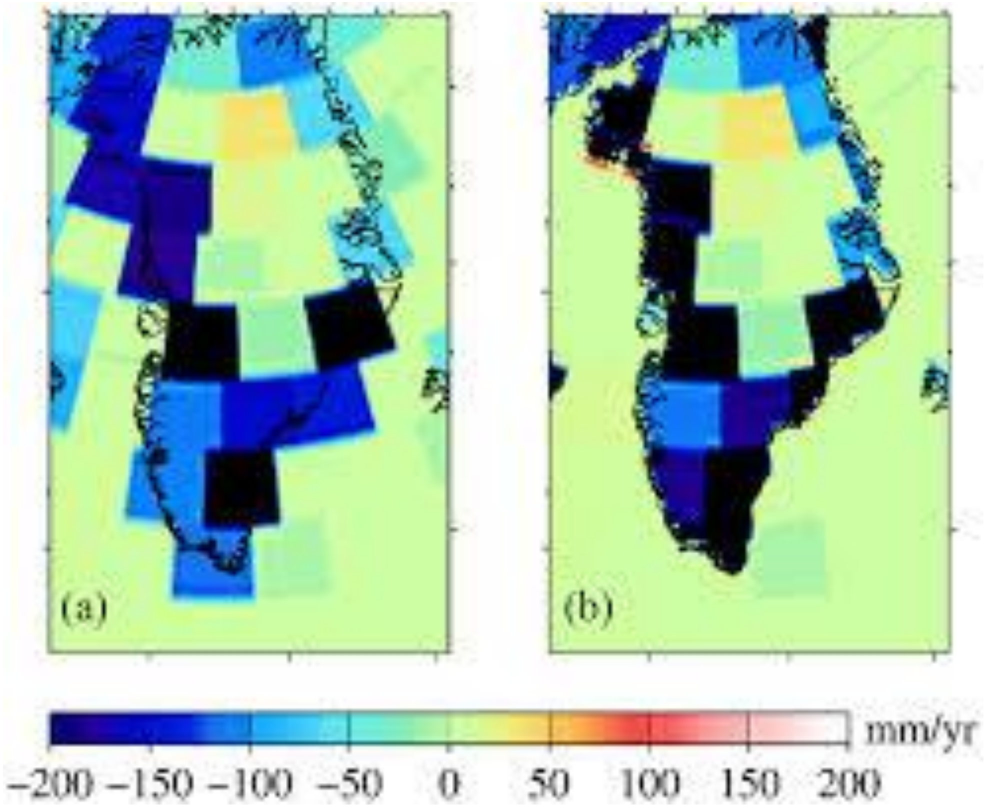
Mascons



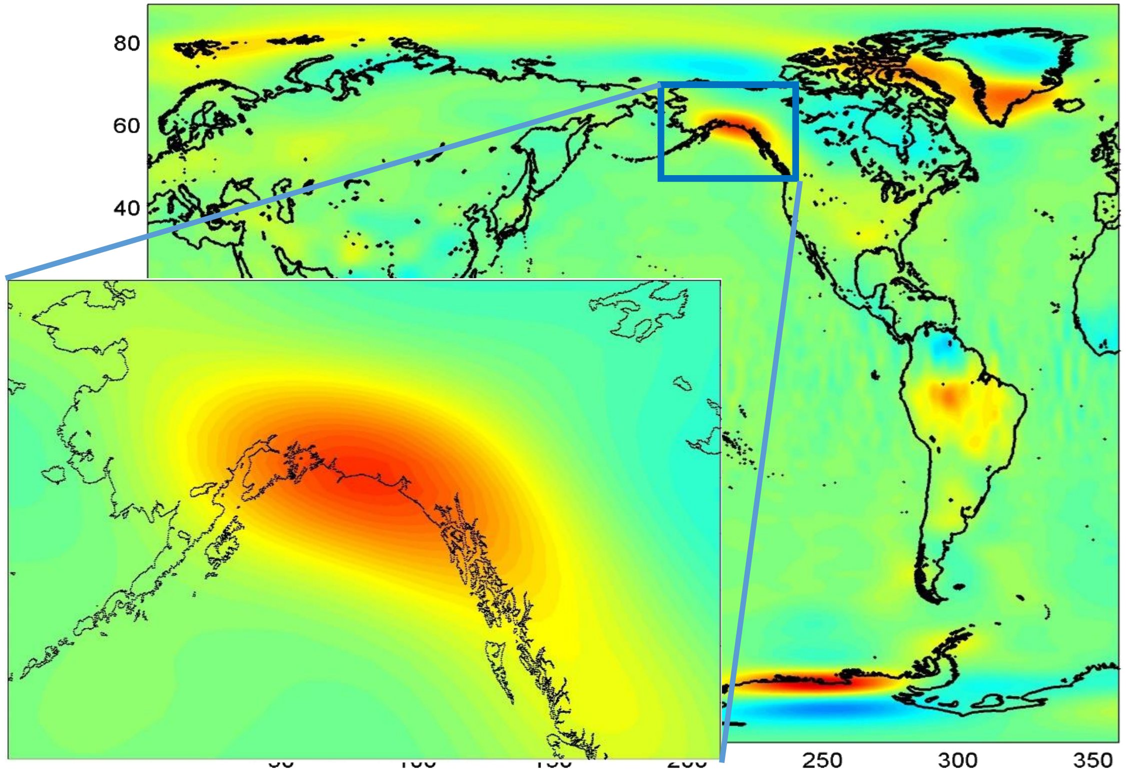
8. Mascons

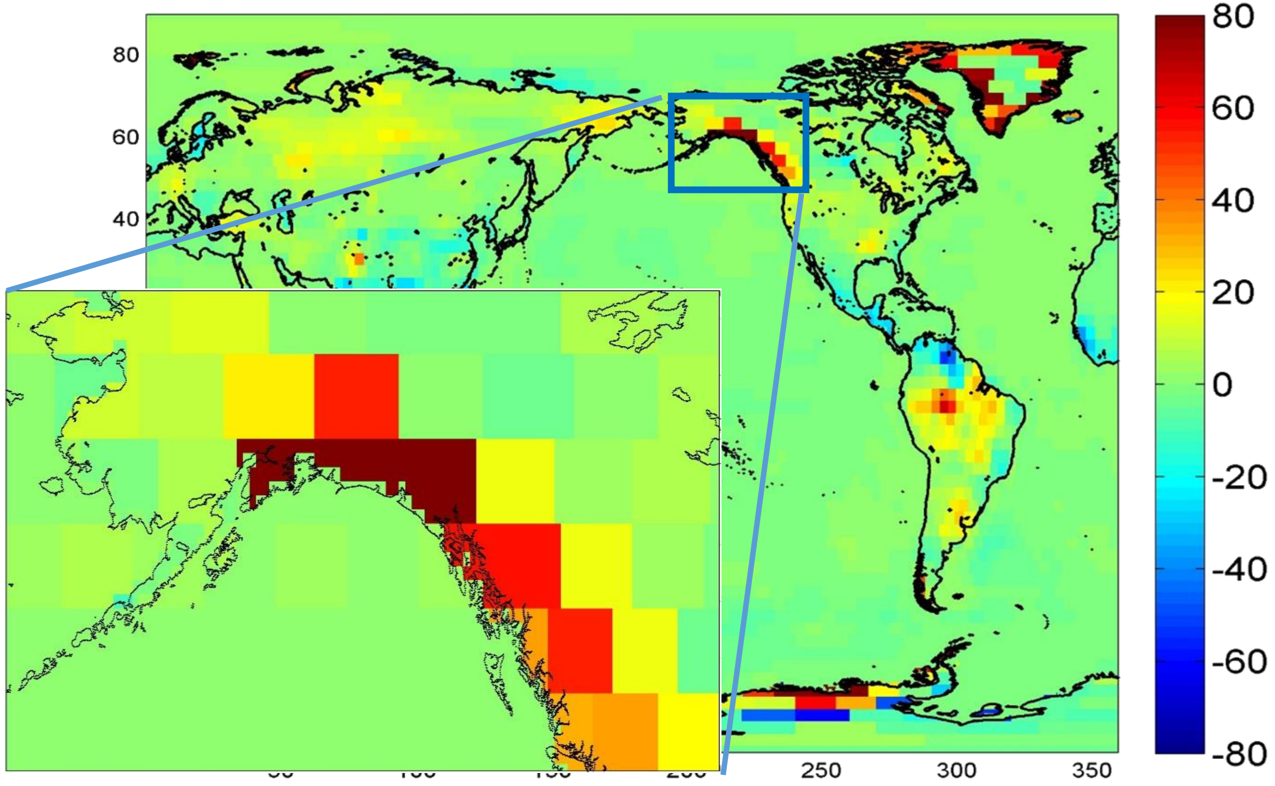
- Instead of SH coefficients (spectral domain), the parameter model consists of mass elements in space domain
- The total effect of all mass elements shall approximate best (in least squares sense) the observations
- Arbitrary mass elements can be used („equivalent source principle“ of potential theory)
    - spheriods/prisms
    - surface area density
    - point masses
    - tesseroids
    - radial base functions

- Additional constraints (e.g. land/ocean, coastlines, ….) can be introduced



8. Spherical Harmonics vs. Mascons





- Parameters in spectral domain
- Unconstrained solution possible
- Leakage-out effects
- Field transformation easy

- Parameters in space domain
- Usually (spatial) constraints: land/ocean, among blocks
- Leakage effects avoided due to space localization
- Field transformation difficult
- More easy-to-use for users ?

Why do we need background models in the gravity field processing ?

E: In order to reduce temporal aliasing effects

F: For the linarization of the observation equation

M: In order to have a reference to verify the final solution

T: To facilitate the interpretation of temporal variation signals

Please guess: What is numerically the most expensive part of gravity field retrieval ?

A: The application of the de-aliasing signals

E: The orbit integration

O: The assembling of the normal equations

U: The inversion of the normal equations

SOLUTION

G

Which observation type contains the highest high-frequency gravity information?

C: satellite altimetry

G: terrestrial gravity

F: satellite gravity: GRACE

H: satellite gravity: GOCE

SOLUTION

G R

Why do we use spherical harmonics for the global representation of Earth's gravity field ?

- **S: Because they are a pain in the ass for many students**
- **A: Because they are orthogonal also in discrete form**
- **R: Because they are a special solution of Laplace equation**
- **T: Because they are stationary and ergodic**

SOLUTION

GRA

What does the maximum degree of the SH expansion physically mean ?

E: There is a lack of base functions

O: It gives the number of zeros in North-South direction

I: It is closely related to the temporal behaviour of the field

A: It determines the maximum spatial resolution of the resulting field

SOLUTION

GRAC

Why is the height of the satellite relevant for the achievable performance ?

C: Because with increasing altitude the high-frequency signals are damped

E: Because in higher altitudes to orbit determination of the satellite is more difficult

D: Because at lower altitudes the increased drag has negative impact on performance

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SOLUTION GRACE F

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SOLUTION

GRACE FO

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SOLUTION

GRACE FO

Take-Home Messages

- Inter-satellite ranging is currently the only method to monitor global mass transport processes on a global scale
- Spherical harmonics are the most commonly used parameterization of the global gravity field. However, alternatives such as Mascons exist.
- Main error sources are temporal aliasing (tidal and non-tidal high-frequency signals), and accelerometer errors.
- In the research unit NEROGRAV we intend to develop advanced processing methods to reduce the impact of these error sources, and to provide more realistic error estimates for the gravity field solutions.

Questions ?