NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

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Lab 1: Spherical harmonic synthesis

Purpose

In this Lab, the technique of spherical harmonic synthesis will be presented. For this purpose, a GRACE-based gravity field model of the Earth up to degree $n = 200$ is provided, which shall then be used to compute various gravity functionals on different altitudes up to varying maximum degrees of expansion. In order to understand the magnitude of temporal changes within the Earth's gravity field as well as some problematics behind the generation of such models (this already poses a transition to Lab 2) a monthly gravity field solution shall be analyzed also.

Material

- Data
  - Static gravity field model *ITSG-Grace2018s* in ICGEM format
  - Gravity field solution for April 2004 *GSM-2_200404_0027* in ICGEM format
  - Coastline file *coast.npz*
- Python functions (found in *SpringSchoolLib12*)
  - `readicgem`
  - `cs_format`
  - `reduce_normpot_grs80`
  - `shs`

Tasks

1. Import the static gravity field model *ITSG-Grace2018s* with `readicgem`. Generate disturbing potential coefficients by reducing the GRS80 normal potential from the given model (`reduce_normpot_grs80`).

2. Compute a global grid of geoid heights up to various maximum harmonic degrees (e.g. $n_{max} = 10, 50, 100, 200$) with `shs`. Plot your results (including coastlines) and interpret them. Experiment with different grid resolutions.
   Hint: For sake of compatibility with the coastline data, please define the longitude vector for `shs` from 0 to 360.

3. Compute gravity disturbances and vertical gravity gradients on a global grid up to various maximum harmonic degrees and visualize your results. How does the signal content change in comparison to the results of task 2? How can this be explained?

4. Compute gravity disturbances up to $n_{max} = 200$ as seen from different satellite altitudes (e.g. $h = 250, 450, 1000, 20000km$). Visualize your results in terms of global grids and interpret them. Based on your findings so far, which conclusions can be drawn for a real gravity field mission?

5. Import the gravity field model of April 2004 (*GSM-2_200404_0027*). Compute the corresponding global geoid heights up to $n_{max} = 90$ analogously to task 1 and 2 and plot the result. Then, compute and visualize this model's deviation from the long-term mean of the gravity field signal (=static field) up to $n_{max} = 90$ (also in terms of geoid heights). Interpret your results.

Python functions:

function [scs, ncs, header, scst, ncst] = readicgem(filename)	
Reads potential coefficients in ICGEM-format from ASCII file	
Input	• `filename`: full path and file name [string]
Output	• `scs`: potential coefficients in cs-format; size [n,n] • `ncs`: formal errors of potential coefficients in cs-format (if available); size [n,n] • `header`: structure containing header information of the ICGEM file • `scst`: dot-coefficients in cs-format (if available); array size [n,n] • `ncst`: formal errors of dot-coefficients in cs-format (if available); size [n,n]
Requires	• `cs_format`

function global_grid = shs(gco, fun, colat, lon, GM, ae, alt)	
Computes a spherical harmonic synthesis of a gravity functional on a global grid	

Input	• `gco`: disturbing potential coefficients given in cs-format; size [n,n]
	• `func`: gravity functional to be computed; list or array
	  – `1`: geoid heights $[m]$
	  – `2`: gravity anomaly $[mGal] = [1^{-5}\ m/s^2]$
	  – `3`: vertical gravity gradient $[E] = [1^{-9}\ m/s^2/m]$
	  – `4`: total water storage $[mm\ EWH] = [kg/m^2]$
	  – `5`: no dimensioning
	  – `6`: gravity disturbance $[mGal] = [1^{-5}\ m/s^2]$
	  – `7`: pressure $[Pa]$
	  – `8`: vertical crustal deformation $[m]$
	• `colat`: co-latitude vector for global grid
	• `lon`: longitude vector for global grid
	• `ae`: radius resp. semi-major axis of Earth $[m]$
	• `alt`: altitude above earth surface for computation of synthesis $[m]$
	• `kwargs`:
	  – `lmax`: maximum degree of expansion
	  – `GM`: gravity constant times Earth mass $[m^3/s^2]$

Output	• `global_grid`: global grid of the computed functional
Requires	• `legnorm` • `loadlove_farrell`

function [scs_model_red] = reduce_normpot_grs80(scs_model, GM, ae)	
Computes disturbing potential coefficients by reducing the GRS80 normal potential from a gravity field model	
Input	• `scs_model`: gravity potential coefficients given in cs-format; size [n,n] • `GM`: gravity constant times Earth mass (ref. to scs_model) $[m^3/s^2]$ • `ae`: radius resp. semi-major axis of Earth (ref. to scs_model)$[m]$
Output	• `scs_model_red`: disturbing potential coefficients in cs-format; size [n,n]
Requires	• `const_grs80`

function [c, s] = cs_format(cs, s)	
Transforms coefficients in cs-format into sc-format or separate c/s matrices and vice versa. cs-format: $\|C\backslash S\|$ sc-format: $\|/SC\backslash\|$ c, s separate: $\|C\backslash\|, \|S\backslash\|$	

Input	• `cs`: if only input, then
	  – size [n,n] implies input in cs-format
	  – size [n,2n] implies input in sc-format
	• `s`: if specified, then cs are c-coefficients while s are s-coefficients, both are of size [n,n]

Output	• `c`: if only output, then
	  – cs-format, i.e. size [n,n] if only input is cs of size [n,2n]
	  – cs-format, i.e. size [n,n] if two inputs are specified
	  – sc-format, i.e. size [n,2n] if only input is cs of size [n,n]
	• `s`: if specified, c contains c-coefficients and s contains s-coefficients, both are of size [n,n]

Requires	• —