NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

International Spring School
Neustadt an der Weinstraße, Germany, March 10-14, 2025

Practical 3: Global analysis of gridded total water storage data

Christian Mielke & Anne Springer (University of Bonn)

berlin

Freie Universität Berlin

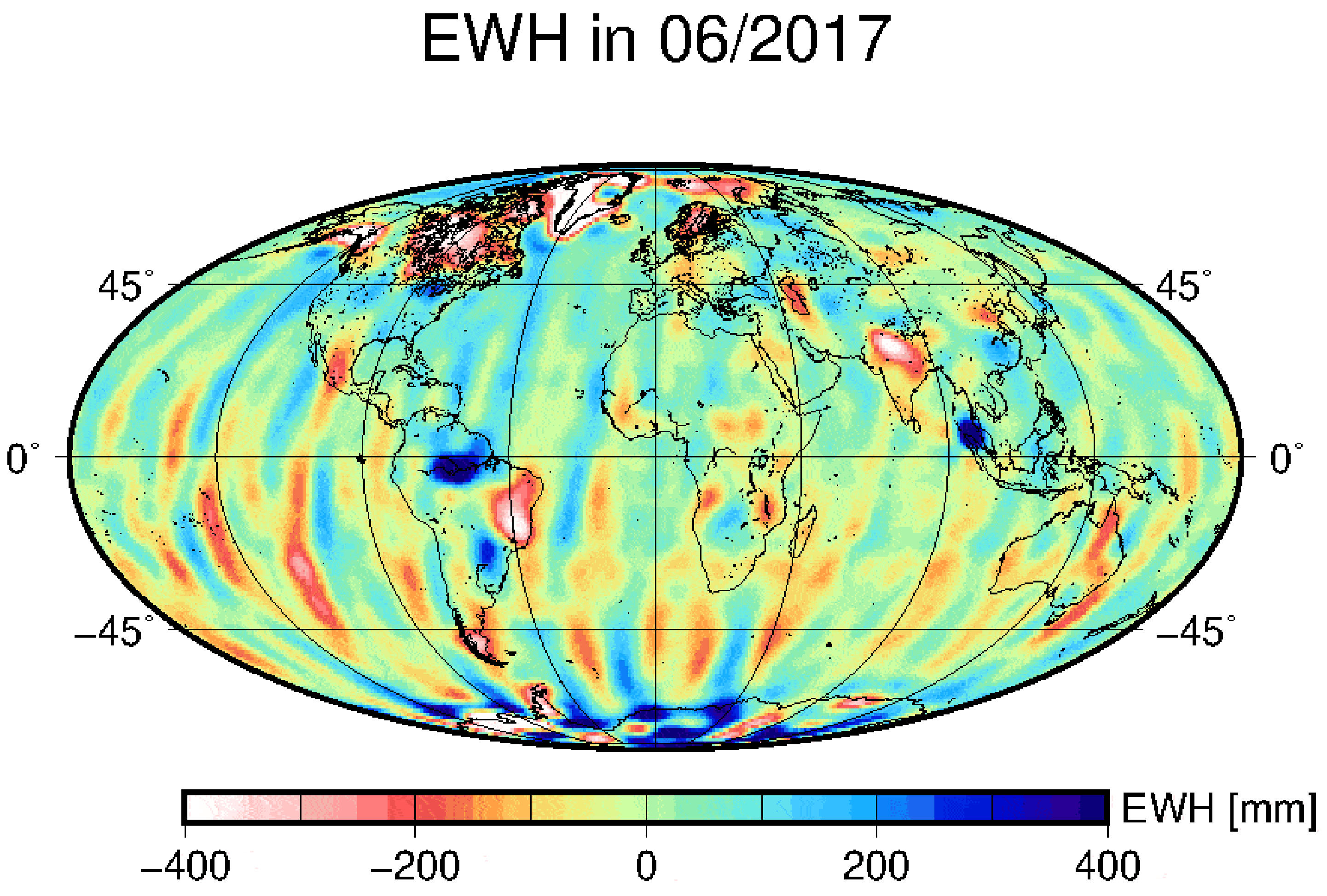
Technische Universität München

GFZ
Helmholtz Centre for Geosciences

UNIVERSITÄT BONN

hcu HafenCity Universität Hamburg

Global analysis of gridded total water storage



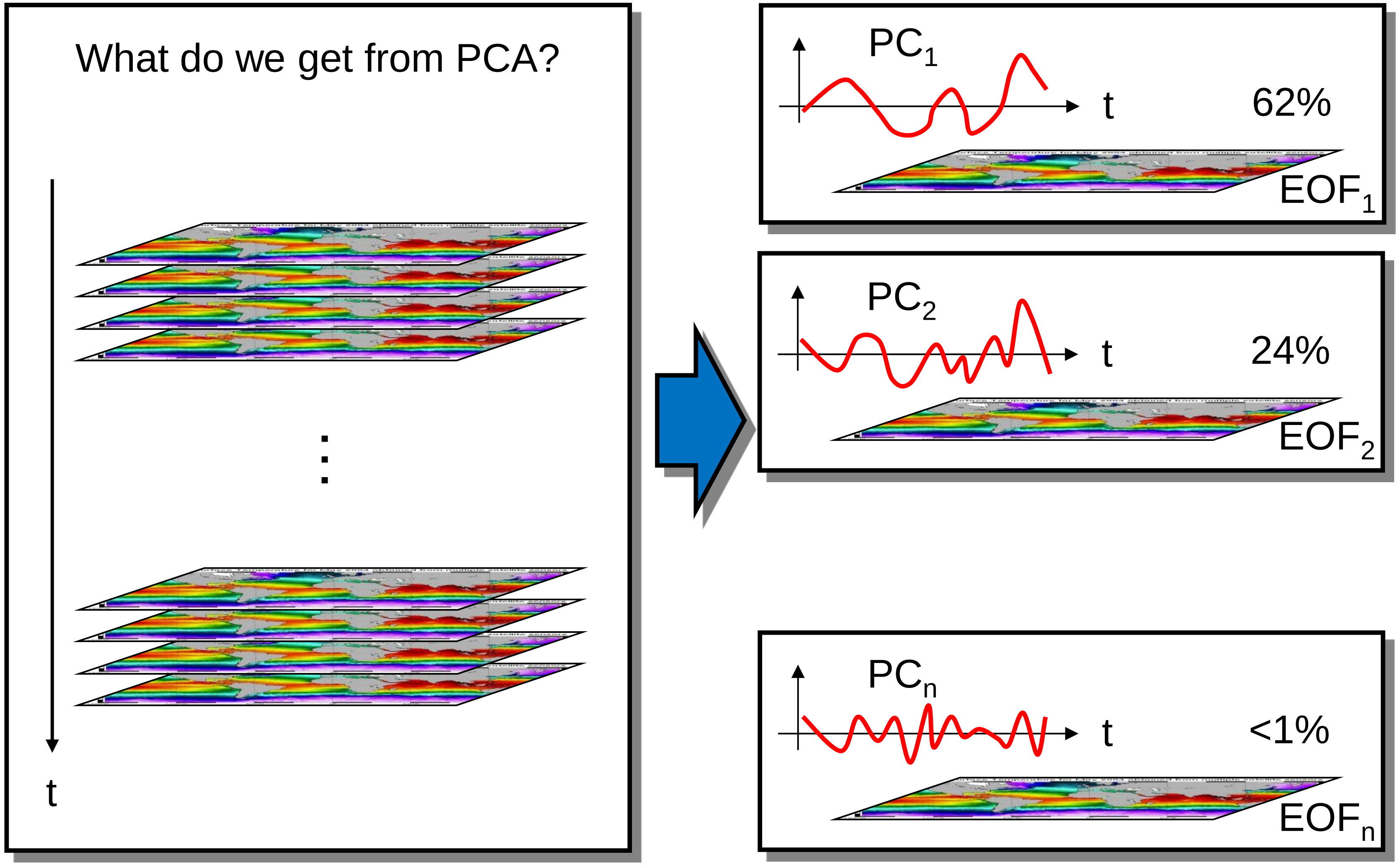
Global analysis of gridded total water storage

Exercise 1: Investigating trends and periodic signals
- Computation of gridded total water storage time series
- Interpretation of geophysical signals

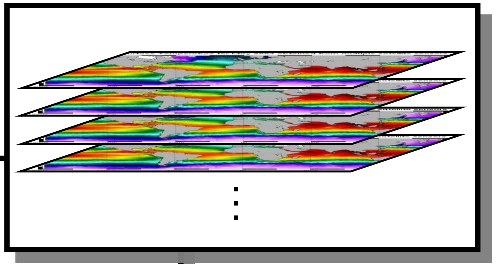
Exercise 2 (optional): Regression against climate indices (Niño 3.4)
- Influence of the El Niño phenomenon on terrestrial water storage

Exercise 3: Principal Component Analysis (PCA)
3a: Calculation and visualization of global EOFs and corresponding PCs
3b: Understanding compression properties of PCA
3c (optional): Understanding domain dependence of PCA

Principal Component Analysis (PCA)



Data matrix



Time series provided as data matrix:

time series for one location

$$\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_p) = \begin{pmatrix} y_{1;1} & y_{1;2} & \cdots & y_{1;p} \\ y_{2;1} & y_{2;2} & \cdots & y_{2;p} \\ \vdots & \vdots & & \vdots \\ y_{n;1} & y_{n;2} & \cdots & y_{n;p} \end{pmatrix}_{n \times p}$$

Ygrace, Ynoah

\# points in time
e.g. 261 months

gridded data for one
point in time,
e.g. one monthly solution

\# locations
e.g. 64.800 grid points

⇨ ***grace_tws.mat, noah_tws.mat***

Eigenvalue decomposition

Calculation of the temporal covariance matrix:

$$\underset{n \times n}{\mathbf{C}} = \frac{1}{p}\mathbf{Y}\mathbf{Y}^{\mathbf{T}}$$

Calculation of the spatial covariance matrix:

$$\underset{p \times p}{\mathbf{C}'} = \frac{1}{p}\mathbf{Y}^{\mathbf{T}}\mathbf{Y}$$

Eigenvalue decomposition of the covariance matrix:

eigenvalues

$$\mathbf{C} = \mathbf{E}\boldsymbol{\Lambda}\mathbf{E}^T = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_n \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{pmatrix} \begin{pmatrix} \mathbf{e}_1^T \\ \mathbf{e}_2^T \\ \vdots \\ \mathbf{e}_n^T \end{pmatrix}$$

eigenvectors

The matrices **C** and **C'** have the same p non-zero eigenvalues.

Eigenvectors and Principal Components

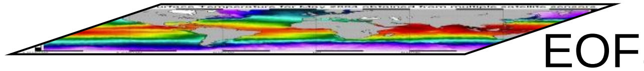
Matrix containing the EOFs:
(normalized, orthogonal basis)

$$\mathbf{E} = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \dots & \mathbf{e}_p \end{pmatrix}_{n \times p}$$

basis functions from GLDAS-NOAH

Sorted according to the size of eigenvalues:

$$\lambda_1 > \lambda_2 > \dots > \lambda_p$$



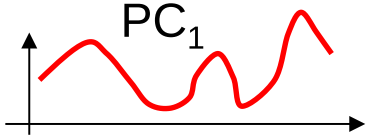
⇨ ***function [eigenvalues, E] = calculateEOF(Y)***

PCs are calculated by projecting the data onto the EOFs: $\mathbf{D} = \mathbf{E}^T\mathbf{Y}$

$$\mathbf{D} = \begin{pmatrix} d_{1;1} & d_{1;2} & \cdots & d_{1;p} \\ d_{2;1} & d_{2;2} & & \\ \vdots & & \ddots & \\ d_{p;1} & & & d_{p;p} \end{pmatrix}_{p \times p}$$

PC_j = temporal evolution of the j-th EOF

d_i scaling factors for time i



⇨ ***function [D] = calculatePC(E,Y)***

Signal Reconstruction

Reconstructed time series: $\mathbf{Y} = \mathbf{E}\mathbf{D}$

Compression: using only the first $\bar{p}$ „major“ EOFs and PCs for the reconstruction as they contain most of the variability.

Signal variability using $\bar{p}$ EOFs and PCs:

$$\mathrm{var} = \frac{\sum_{j=1}^{\bar{p}} \lambda_j}{\Delta^2} \qquad \text{with} \qquad \Delta^2 = \sum_{j=1}^{p} \lambda_j$$

(total variance)

NEROGRAV

New Refined Observations of Climate Change from Spaceborne Gravity Missions

Have a lot of fun and do not hesitate to ask questions!

TU berlin | Freie Universität Berlin | Technische Universität München | GFZ Helmholtz Centre for Geosciences | Universität Bonn | hcu HafenCity Universität Hamburg